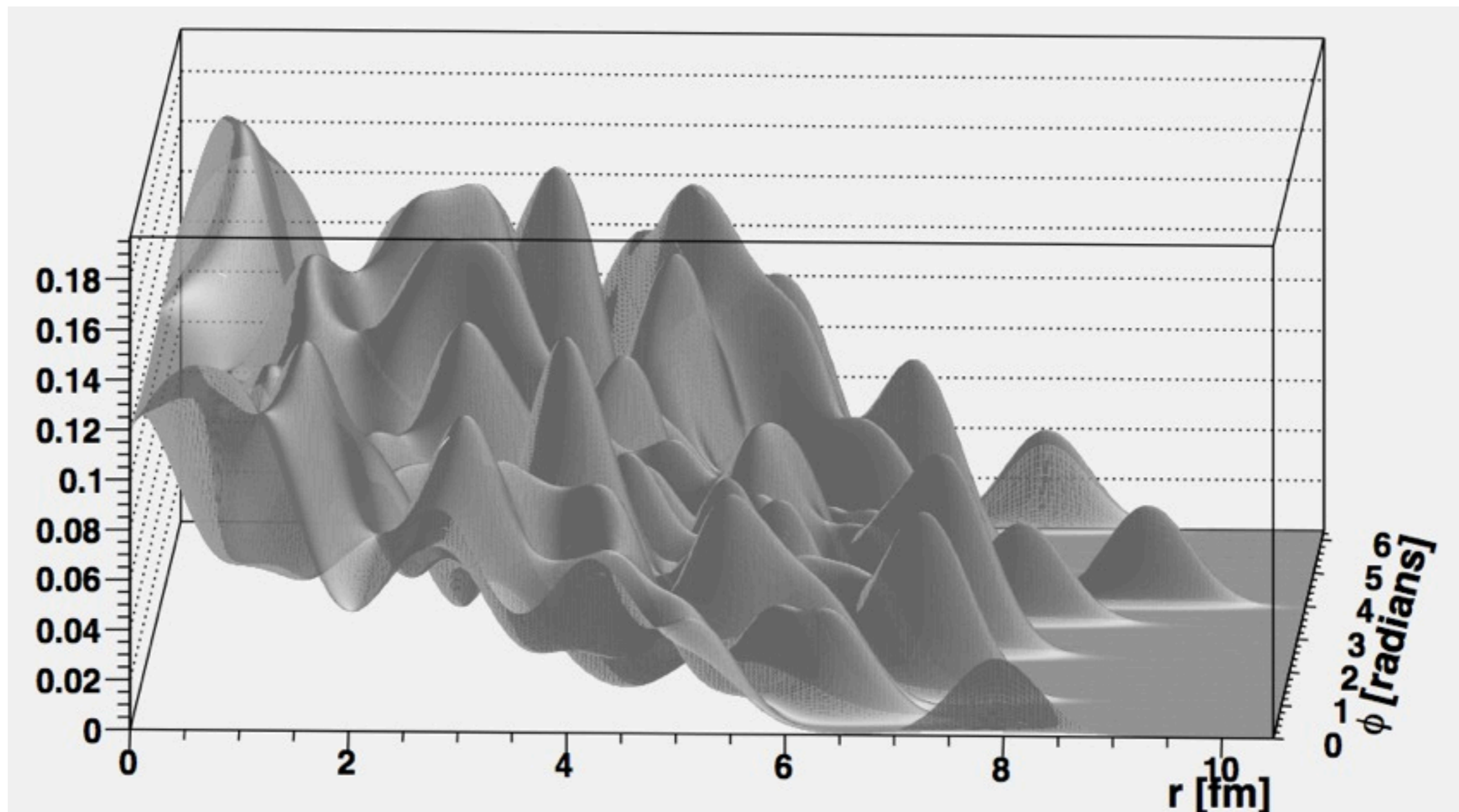


Sartre

a new Monte Carlo for small x diffraction at an Electron Ion Collider

DIS 2011

Tobias Toll



Two Key Measurements at an EIC in eA

Small x /Saturation:

Distribution of gluon
momenta within the nucleus

$$F_2^A \text{ \& \& } F_L^A$$

Distribution of gluon
positions within the nucleus
Measured with exclusive diffractive vector
meson production and DVCS

A Monte Carlo for eA scattering

There is a vast number of generators for
 ep scattering.

For eA scatterings the situation is much less
favourable.

For eA scatterings at an EIC, $\sim 30\%$ of the
events are expected to be diffractive.

No Monte Carlo generator exists for
describing these events.

A Monte Carlo for eA scattering

There is a vast number of generators for
 ep scattering.

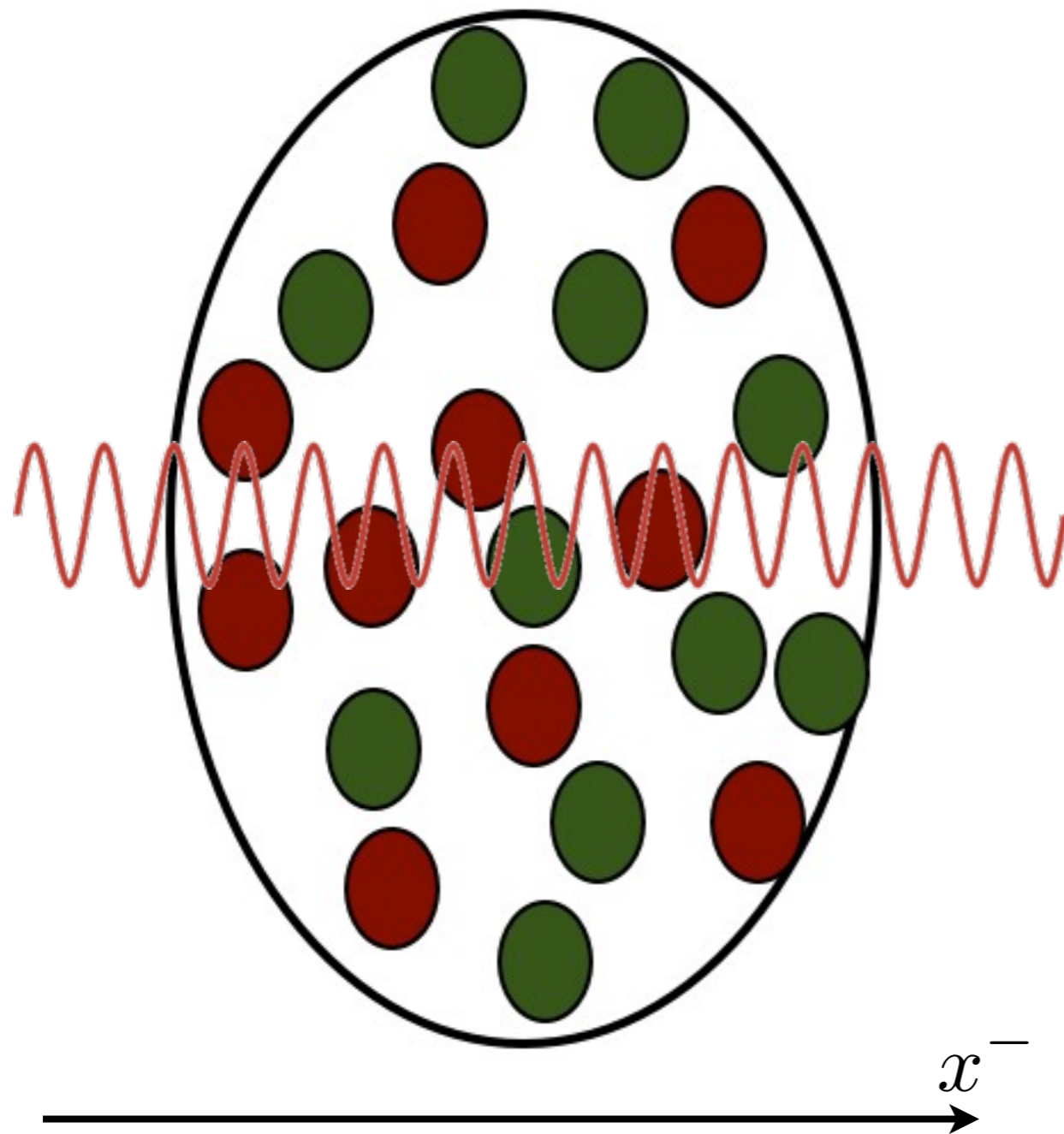
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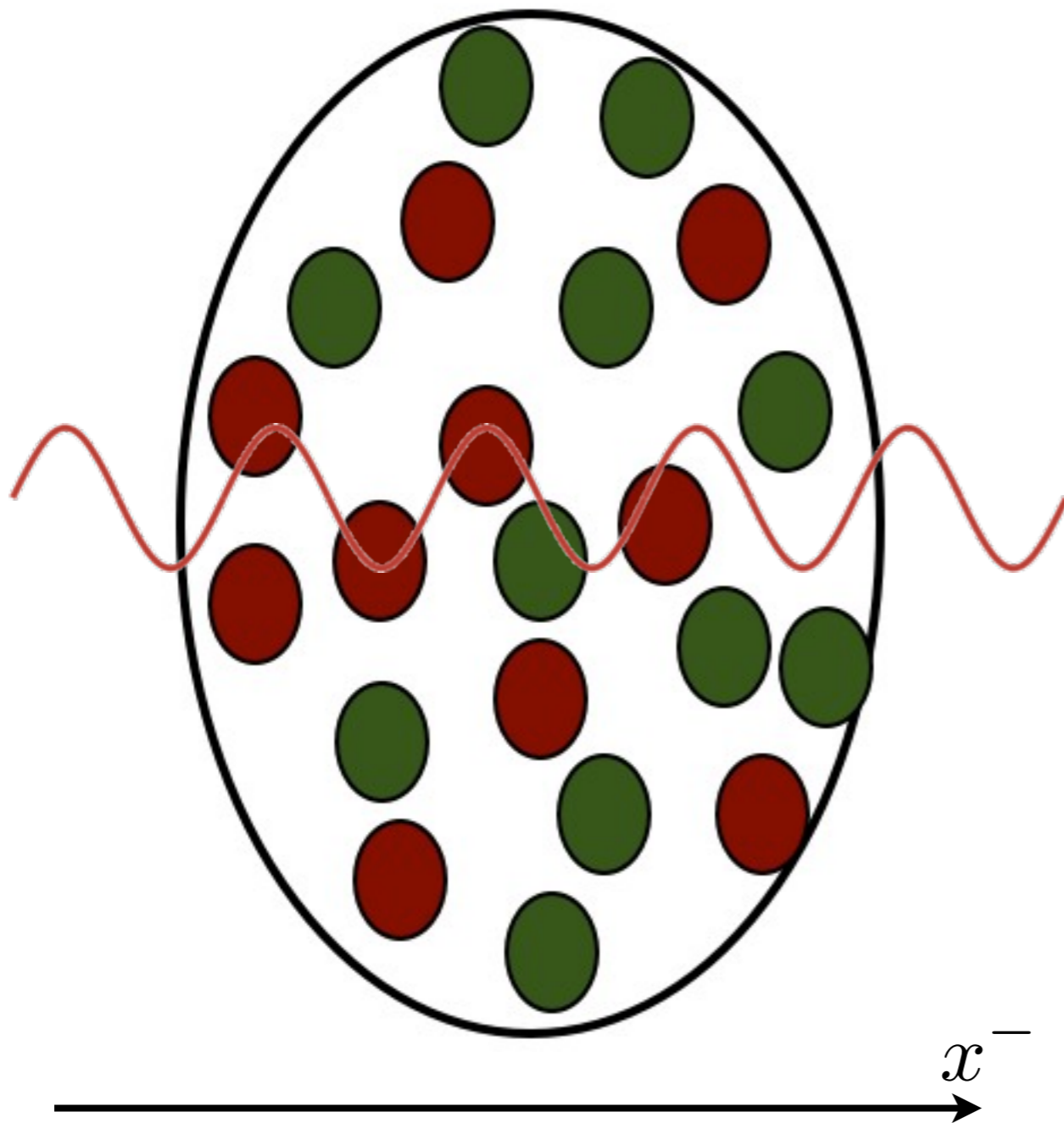
Until Now!

Probing the Nucleus at small x



At large x : large p^+ ,
short wavelength in x^- ,
individual nucleons
can be resolved.

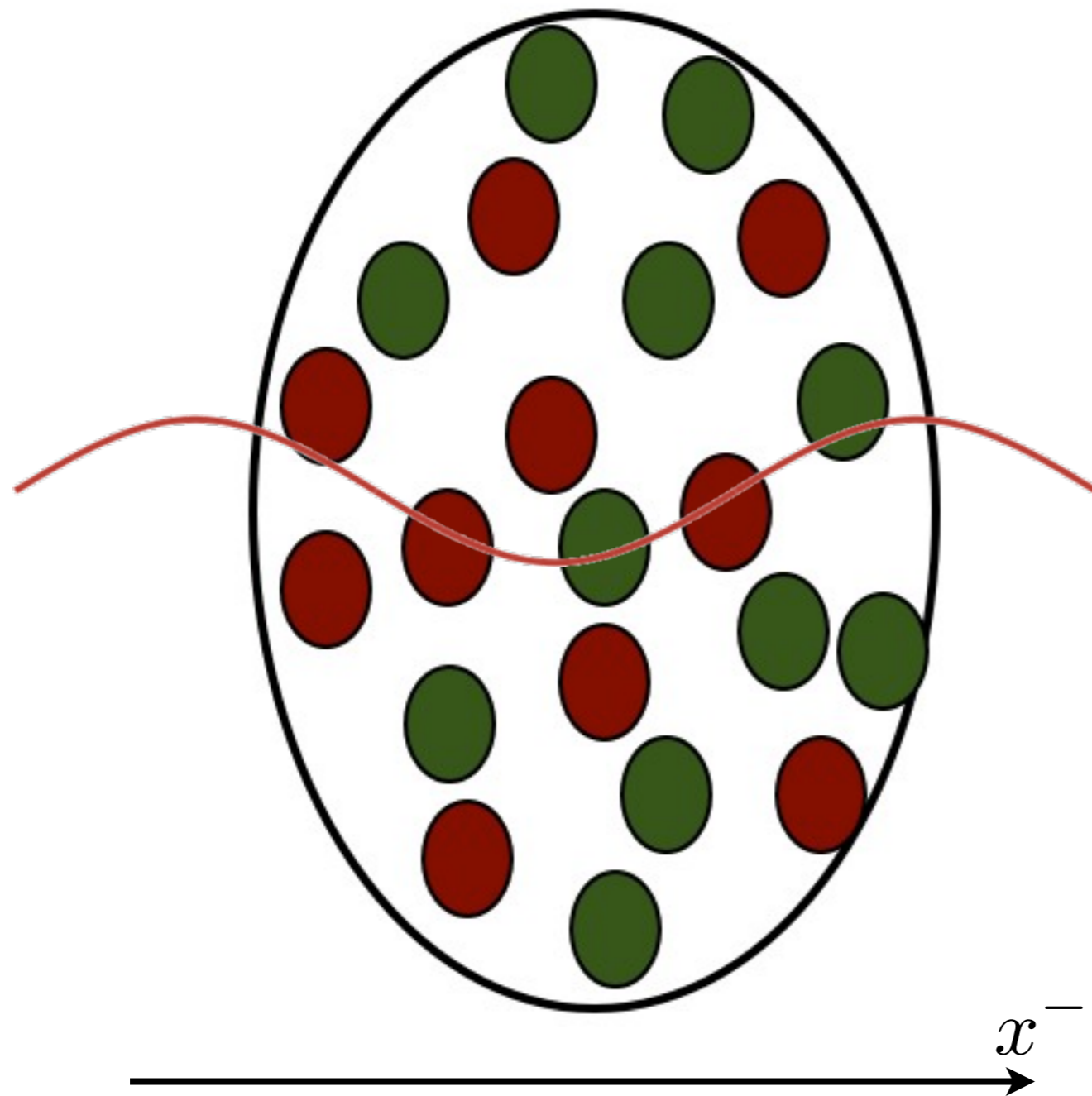
Probing the Nucleus at small x



At large x : large p^+ ,
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At smaller x ,
coherently probe larger area.

Probing the Nucleus at small x



At large x : large p^+ ,
short wavelength in x^- ,
individual nucleons
can be resolved.

At smaller x ,
coherently probe larger area.

At $x \ll \frac{A^{-1/3}}{M_N R_p}$
coherently probing
the whole nucleus.

Challenge for MC, can not just use “A x Pythia”!!

A Monte Carlo for eA scattering

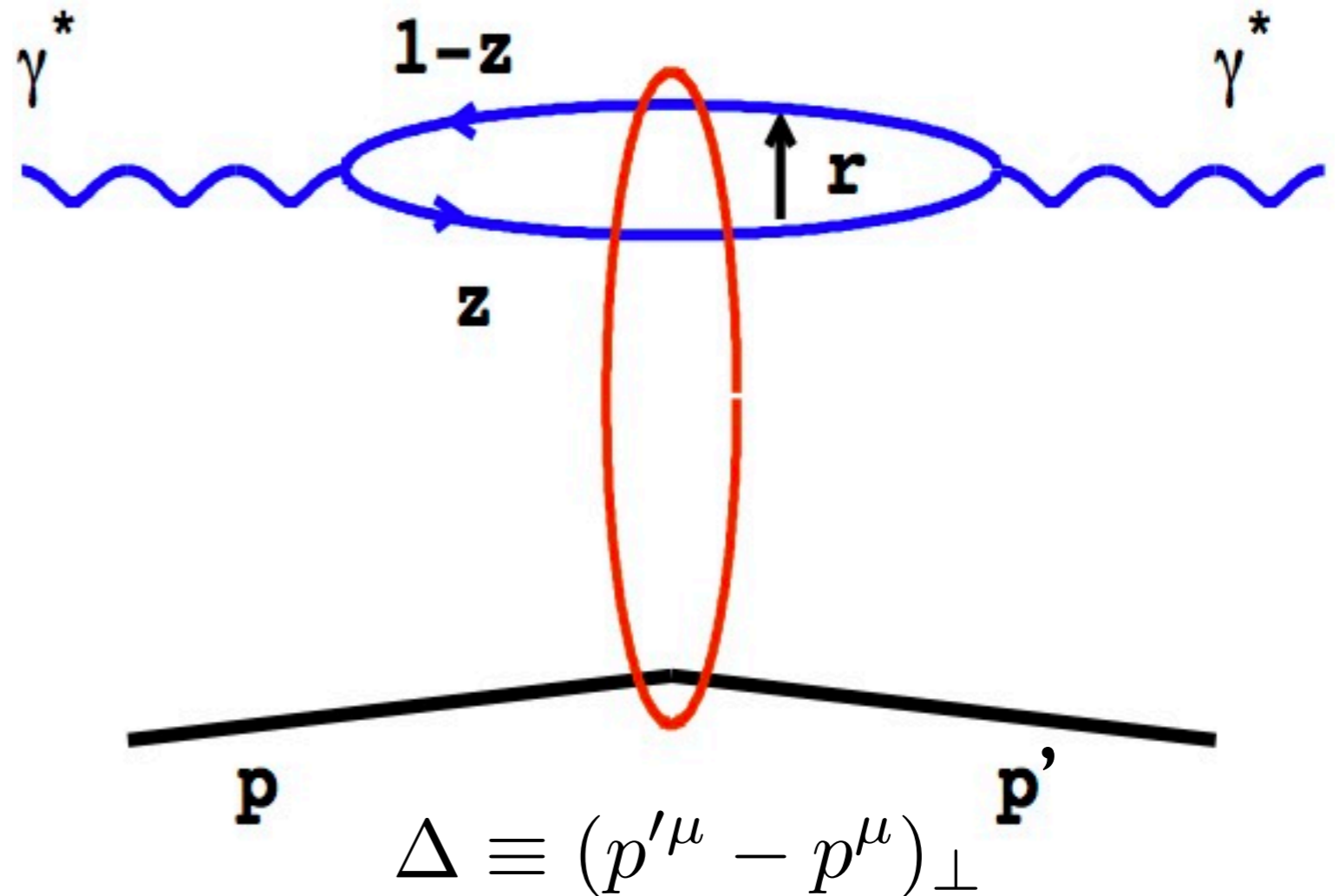
To simulate a measurement of spacial gluon distribution in a nucleus we need a Monte Carlo for eA which describes:

Exclusive Diffractive Vector Meson production
and DVCS
at
Small x

Start with *ep*

The Dipole Model

Elastic photon-proton
scattering



$$\mathcal{A}^{\gamma^* p}(x, Q, \Delta) = \sum_f \sum_{h, \bar{h}} \int d^2 \mathbf{r} \int_0^1 \frac{dz}{4\pi} \Psi_{h\bar{h}}^*(r, z, Q) \mathcal{A}_{q\bar{q}}(x, r, \Delta) \Psi_{h\bar{h}}(r, z, Q)$$

Exclusive diffractive processes at HERA within the dipole picture, H. Kowalski, L. Motyka, G. Watt, Phys. Rev. D74, 074016,
arXiv:[hep-ph/0606272v2](https://arxiv.org/abs/hep-ph/0606272v2)

The Dipole Model

$$\mathcal{A}^{\gamma^* p}(x, Q, \Delta) = \sum_f \sum_{h, \bar{h}} \int d^2 \mathbf{r} \int_0^1 \frac{dz}{4\pi} \Psi_{h\bar{h}}^*(r, z, Q) \mathcal{A}_{q\bar{q}}(x, r, \Delta) \Psi_{h\bar{h}}(r, z, Q)$$

Use:

Optical theorem:

$$\mathcal{A}_{q\bar{q}}(x, r, \Delta) = \int d^2 \mathbf{b} e^{-i\mathbf{b} \cdot \Delta} \mathcal{A}_{q\bar{q}}(x, r, b) = i \int d^2 \mathbf{b} e^{-i\mathbf{b} \cdot \Delta} 2 [1 - S(x, r, b)] .$$

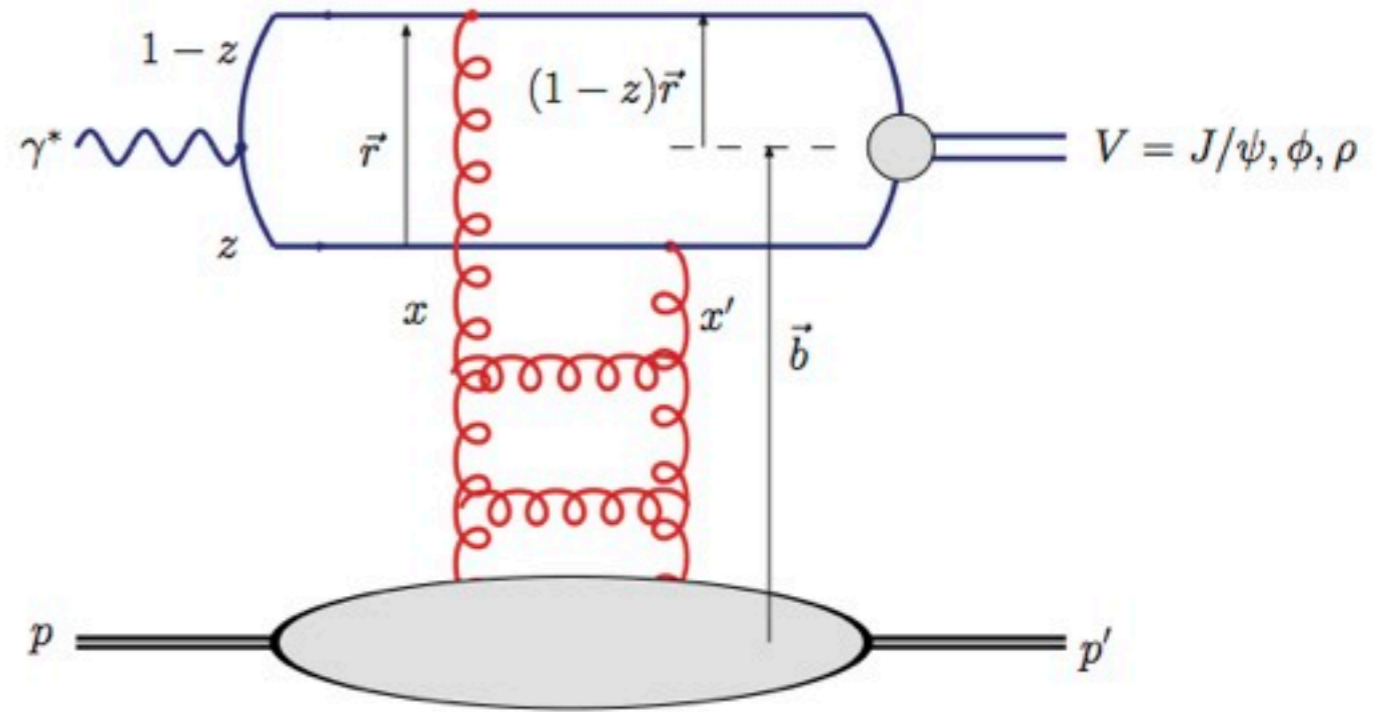
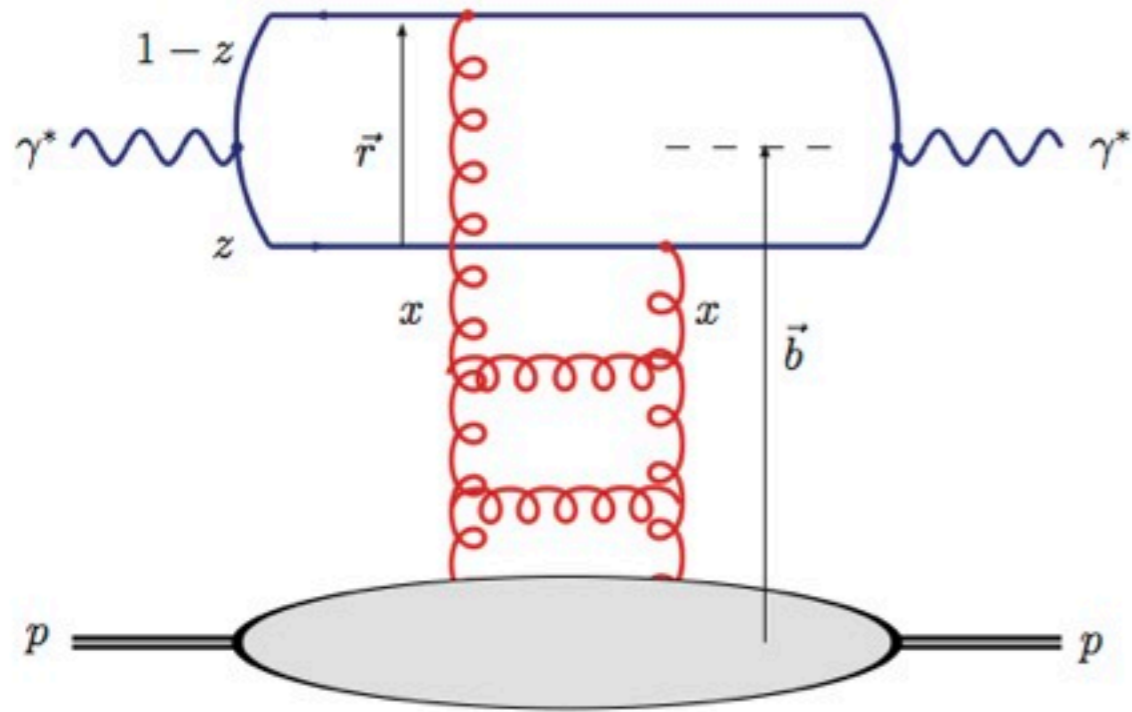
Real Part of S-matrix:

$$\sigma_{q\bar{q}}(x, r) = \text{Im } \mathcal{A}_{q\bar{q}}(x, r, \Delta = 0) = \int d^2 \mathbf{b} 2[1 - \text{Re } S(x, r, b)]$$

$\mathcal{N}(x, r, b)$

Define dipole cross-section: $\frac{d\sigma_{q\bar{q}}}{d^2 \mathbf{b}} = 2\mathcal{N}(x, r, b)$

Vector Meson Production



$$\mathcal{A}_{T,L}^{\gamma^* p \rightarrow V p}(x, Q, \Delta) =$$

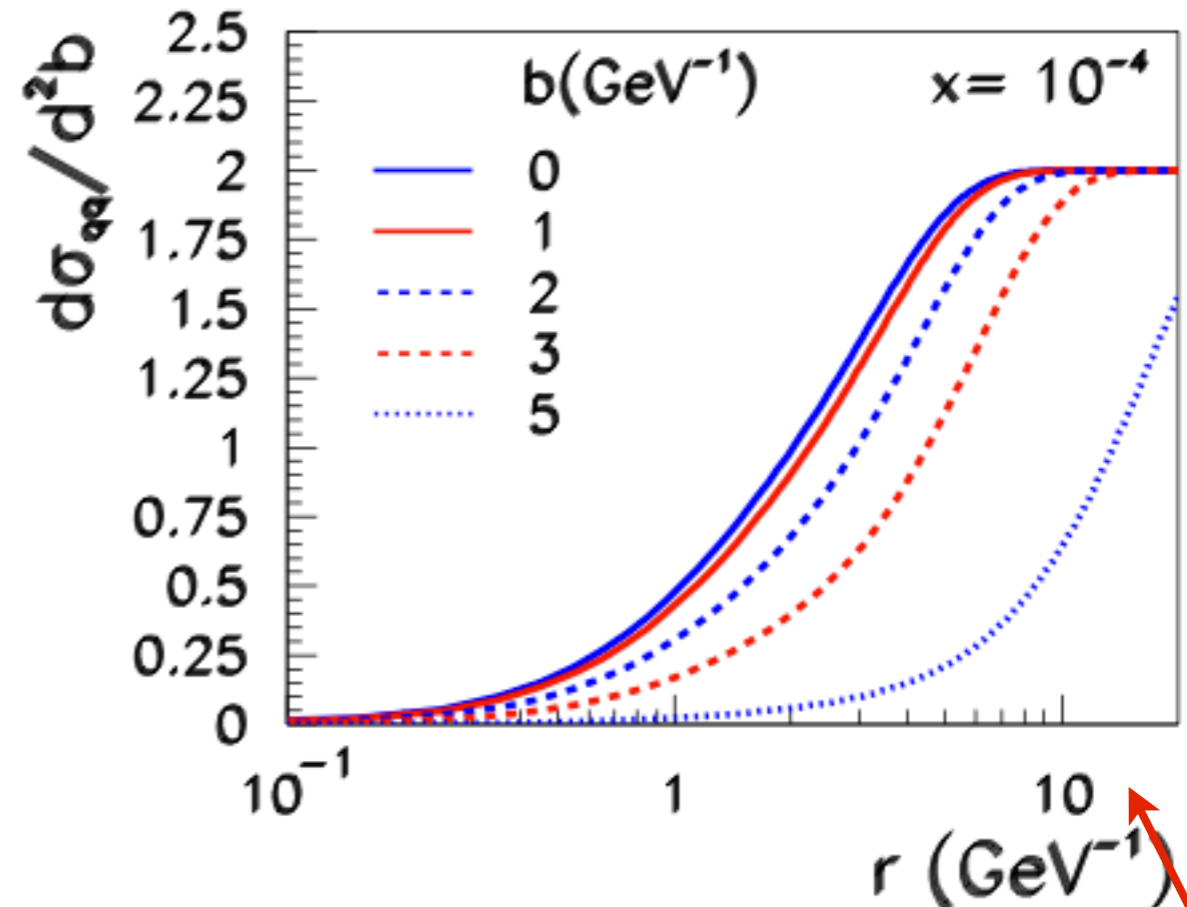
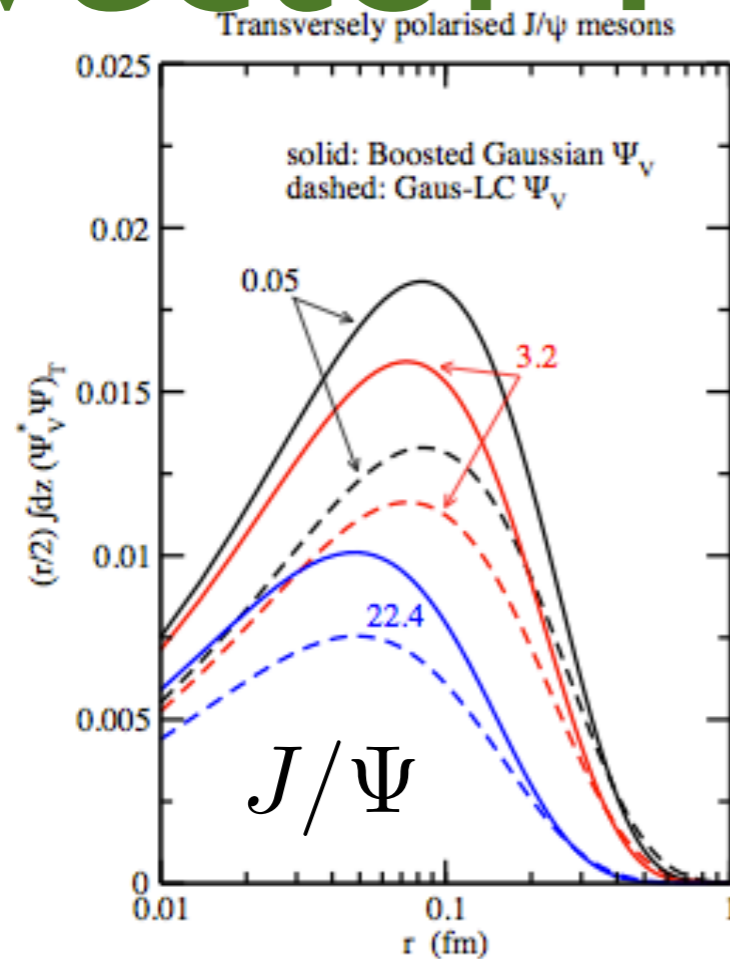
$$i \int d^2 \mathbf{r} \int_0^1 \frac{dz}{4\pi} \int d^2 \mathbf{b} (\Psi_V^* \Psi)_{T,L} e^{-i([1-z]\mathbf{r} + \mathbf{b}) \cdot \Delta} \frac{d\sigma_{q\bar{q}}}{d^2 \mathbf{b}}$$

$$\Delta \equiv (p'^\mu - p^\mu)_\perp$$

Known from QED

Needs to be modeled

Vector Meson Production



$$\mathcal{A}_{T,L}^{\gamma^* p \rightarrow V p}(x, Q, \Delta) =$$

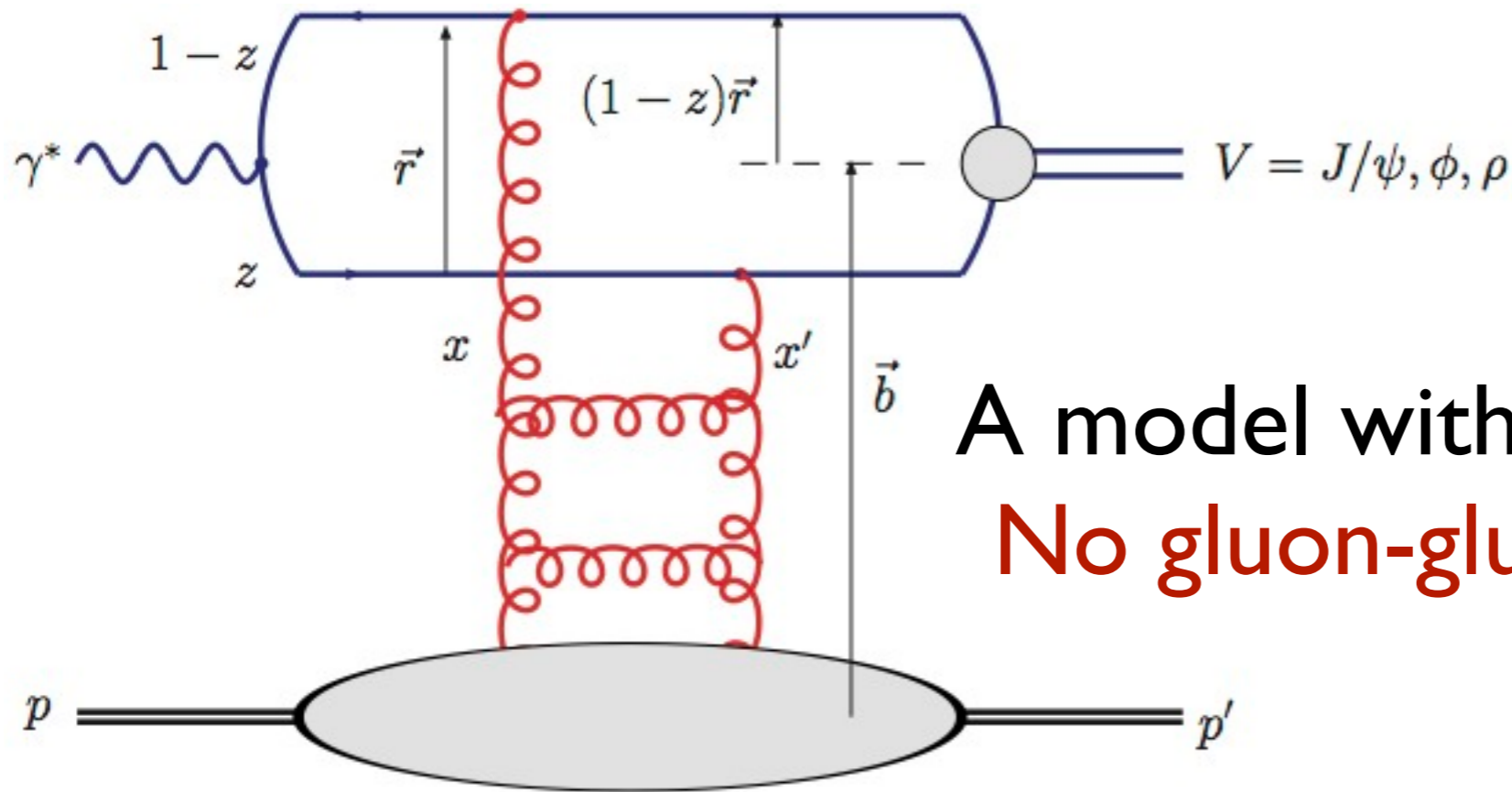
$$i \int d^2\mathbf{r} \int_0^1 \frac{dz}{4\pi} \int d^2\mathbf{b} (\Psi_V^* \Psi)_{T,L} e^{-i([1-z]\mathbf{r} + \mathbf{b}) \cdot \Delta} \frac{d\sigma_{q\bar{q}}}{d^2\mathbf{b}}$$

Known from QED

$$\Delta \equiv (p'^\mu - p^\mu)_\perp$$

Needs to be modeled

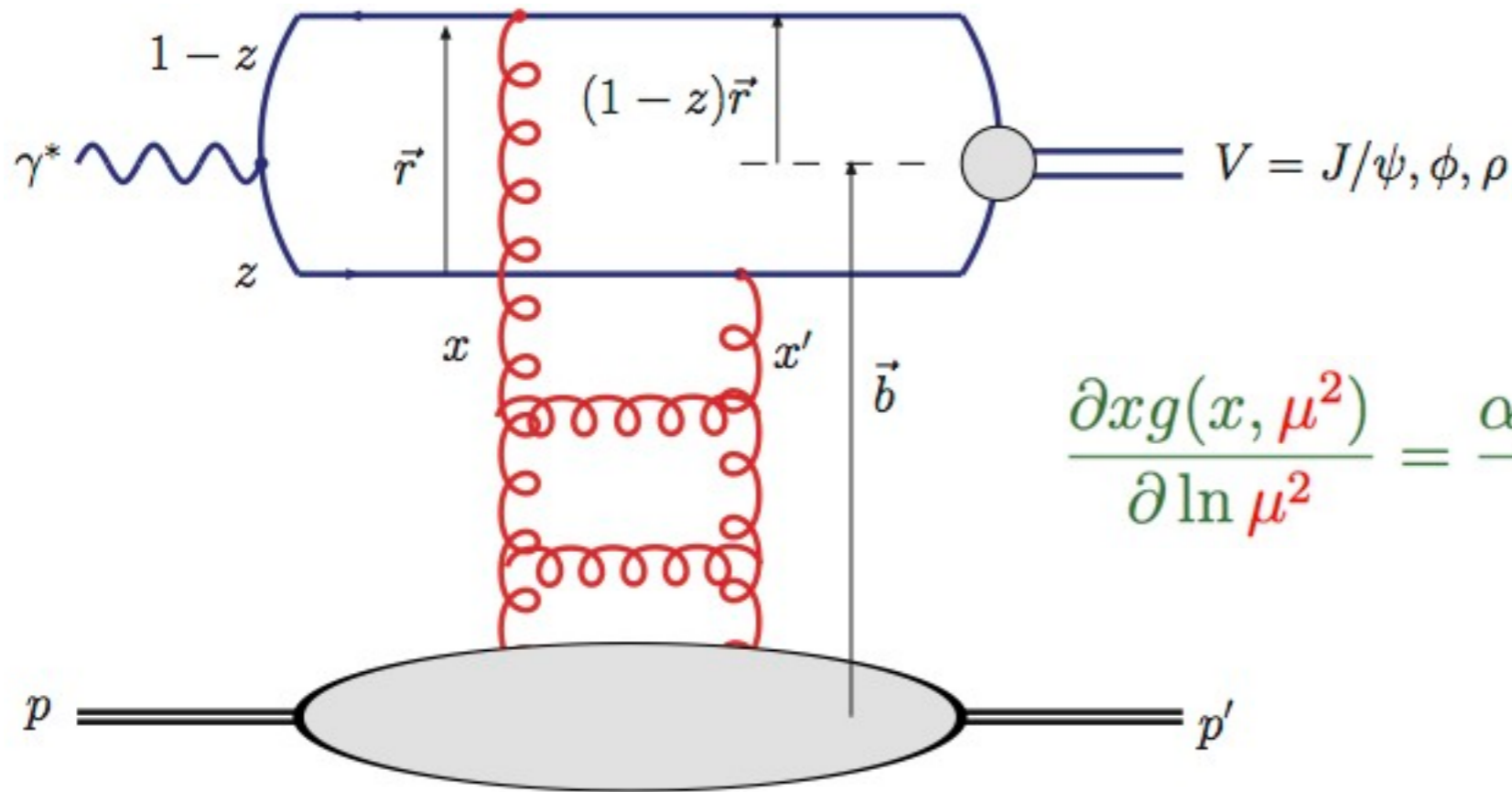
The b-Sat Model



A model with multiple scatterings.
No gluon-gluon recombinations!

$$\frac{d\sigma_{q\bar{q}}}{d^2\mathbf{b}} = 2 \left[1 - \exp \left(-\frac{\pi^2}{2N_c} r^2 \alpha_s(\mu^2) x g(x, \mu^2) T(b) \right) \right]$$

The b-Sat Model

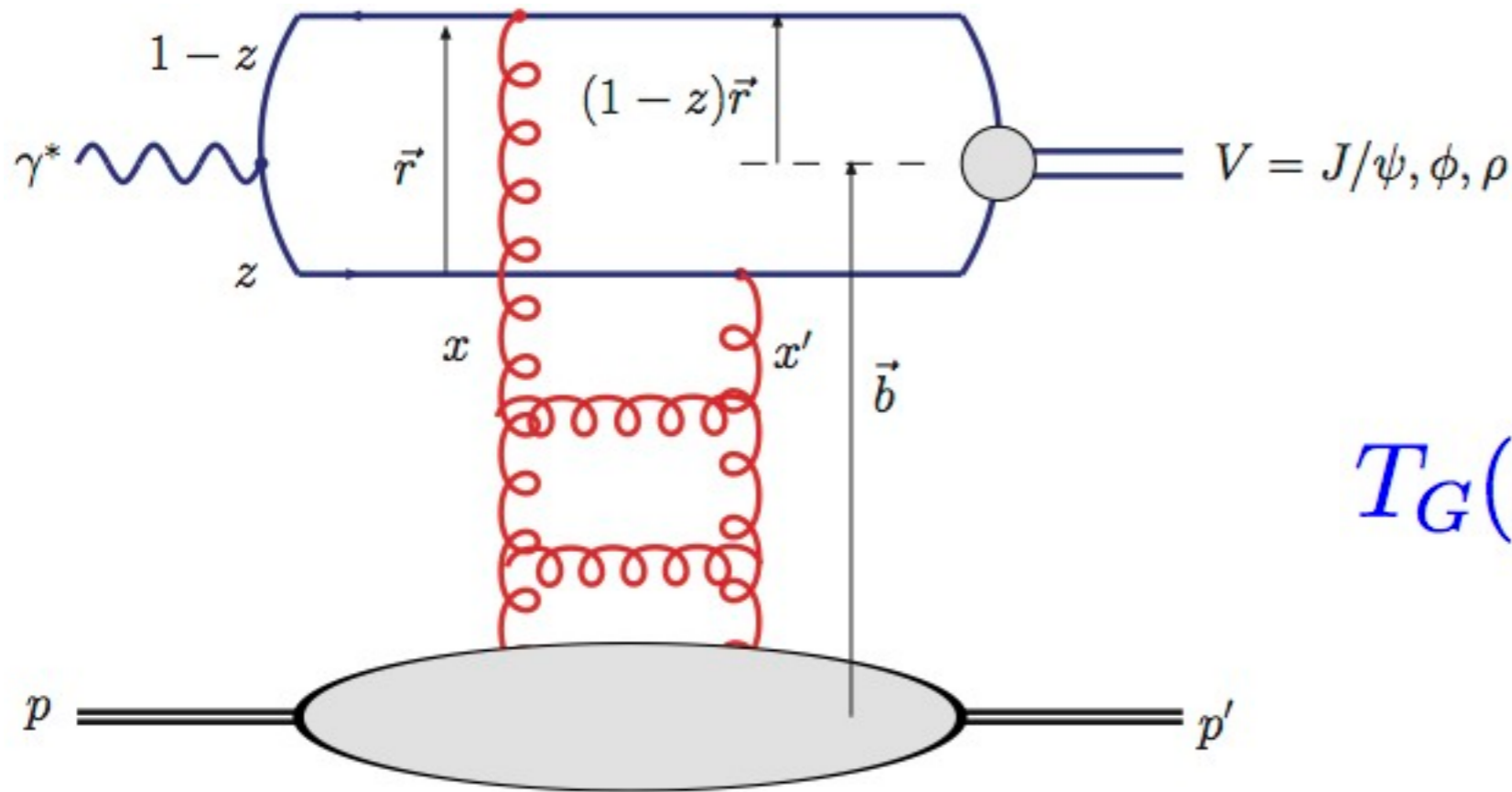


$$\frac{\partial x g(x, \mu^2)}{\partial \ln \mu^2} = \frac{\alpha_s(\mu^2)}{2\pi} \int_x^1 dz P_{gg}(z) \frac{x}{z} g\left(\frac{x}{z}, \mu^2\right)$$

$$\frac{d\sigma_{q\bar{q}}}{d^2\mathbf{b}} = 2 \left[1 - \exp \left(-\frac{\pi^2}{2N_c} r^2 \alpha_s(\mu^2) x g(x, \mu^2) T(b) \right) \right]$$

$$\mu^2 = \frac{4}{r^2} + \mu_0^2$$

The b-Sat Model

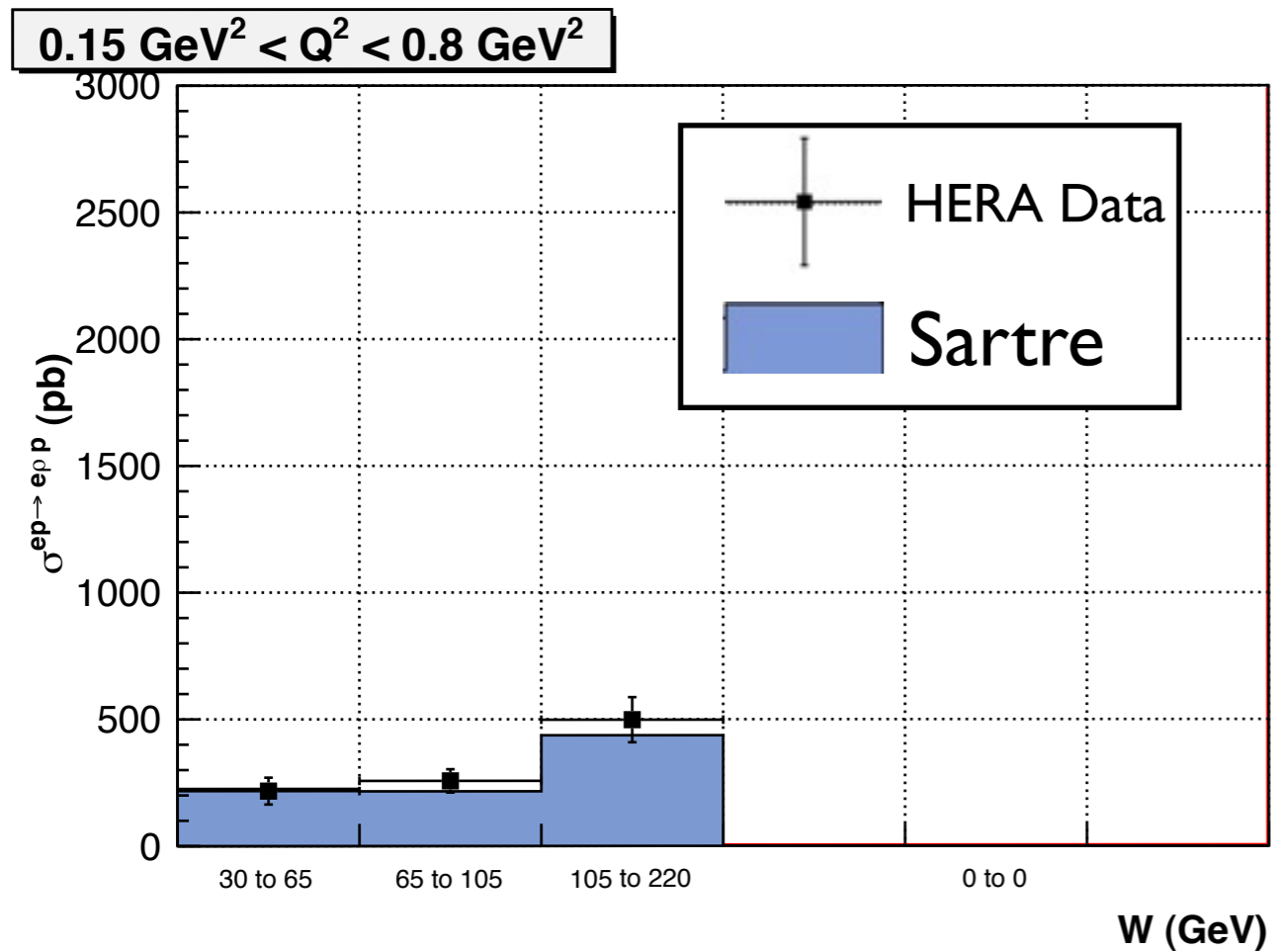


$$T_G(b) = \frac{1}{2\pi B_G} e^{-\frac{b^2}{2B_G}}$$

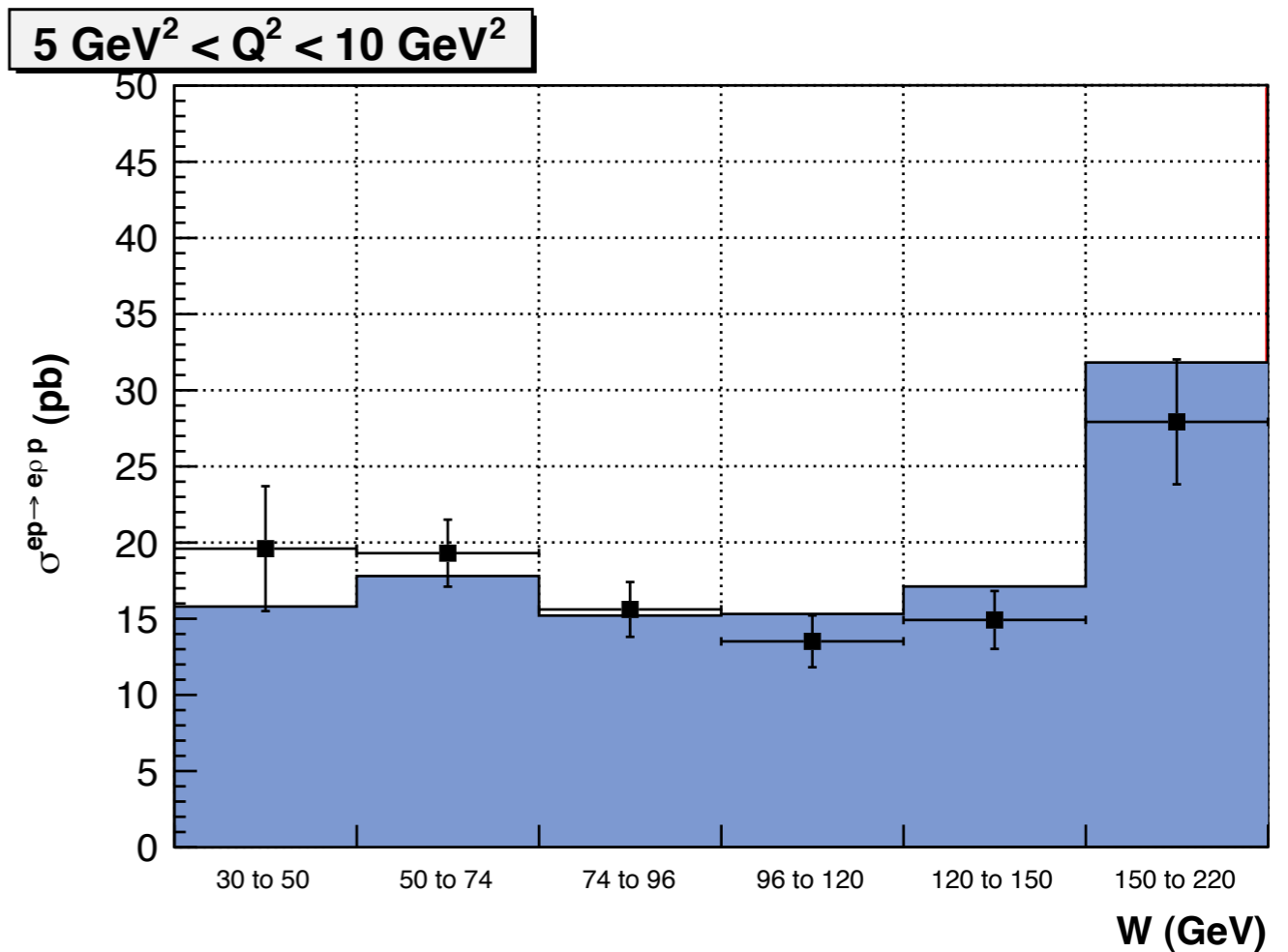
$$\frac{d\sigma_{q\bar{q}}}{d^2\mathbf{b}} = 2 \left[1 - \exp \left(-\frac{\pi^2}{2N_c} r^2 \alpha_s(\mu^2) x g(x, \mu^2) T(b) \right) \right]$$

J/Ψ at HERA vs. b-Sat

Exclusive electroproduction of J/Ψ mesons at HERA Nuc. Phys. B695



Plots produced by M. Savastio



Going from ep to eA

Going from ep to eA

ep :

$$\text{Re}(S) = 1 - \mathcal{N}^{(p)}(x, r, \mathbf{b}) = 1 - \frac{1}{2} \frac{d\sigma_{q\bar{q}}^{(p)}(x, r, \mathbf{b})}{d^2\mathbf{b}}$$

eA :

$$1 - \mathcal{N}^{(A)} = \prod_{i=1}^A \left(1 - \mathcal{N}^{(p)}(x, r, |\mathbf{b} - \mathbf{b}_i|) \right)$$

Assume the Wood-Saxon distribution



$b\text{Sat}$:

$$\frac{d\sigma_{q\bar{q}}^A}{d^2\mathbf{b}} = 2 \left[1 - \exp \left(-\frac{\pi^2}{2N_c} r^2 \alpha_s(\mu^2) x g(x, \mu^2) \sum_{i=1}^A T_p(\mathbf{b} - \mathbf{b}_i) \right) \right]$$

Going from ep to eA

Another difference in eA :
The Nucleus can break up
into colour neutral fragments!

When the nucleus breaks up, the scattering is called
incoherent

When the nucleus stays intact, the scattering is called
coherent

Total cross-section = **incoherent** + **coherent**

Incoherent Scattering

Nucleus dissociates ($f \neq i$):

Good, Walker

$$\begin{aligned}
 \sigma_{\text{incoherent}} &\propto \sum_{f \neq i} \langle i | \mathcal{A} | f \rangle^\dagger \langle f | \mathcal{A} | i \rangle && \text{complete set} \\
 &= \sum_f \langle i | \mathcal{A} | f \rangle^\dagger \langle f | \mathcal{A} | i \rangle - \langle i | \mathcal{A} | i \rangle^\dagger \langle i | \mathcal{A} | i \rangle \\
 &= \langle i | |\mathcal{A}|^2 | i \rangle - |\langle i | \mathcal{A} | i \rangle|^2 = \langle |\mathcal{A}|^2 \rangle - |\langle \mathcal{A} \rangle|^2
 \end{aligned}$$

The incoherent CS is the variance of the amplitude!!

$$\frac{d\sigma_{\text{total}}}{dt} = \frac{1}{16\pi} \langle |\mathcal{A}|^2 \rangle$$

$$\frac{d\sigma_{\text{coherent}}}{dt} = \frac{1}{16\pi} |\langle \mathcal{A} \rangle|^2$$

Averaging over initial states

$$\frac{d\sigma_{\text{total}}}{dt} = \frac{1}{16\pi} \left\langle |\mathcal{A}|^2 \right\rangle_{\Omega} \quad \frac{d\sigma_{\text{coherent}}}{dt} = \frac{1}{16\pi} |\langle \mathcal{A} \rangle_{\Omega}|^2$$

The average should be taken over initial nucleon configurations Ω within the nucleus

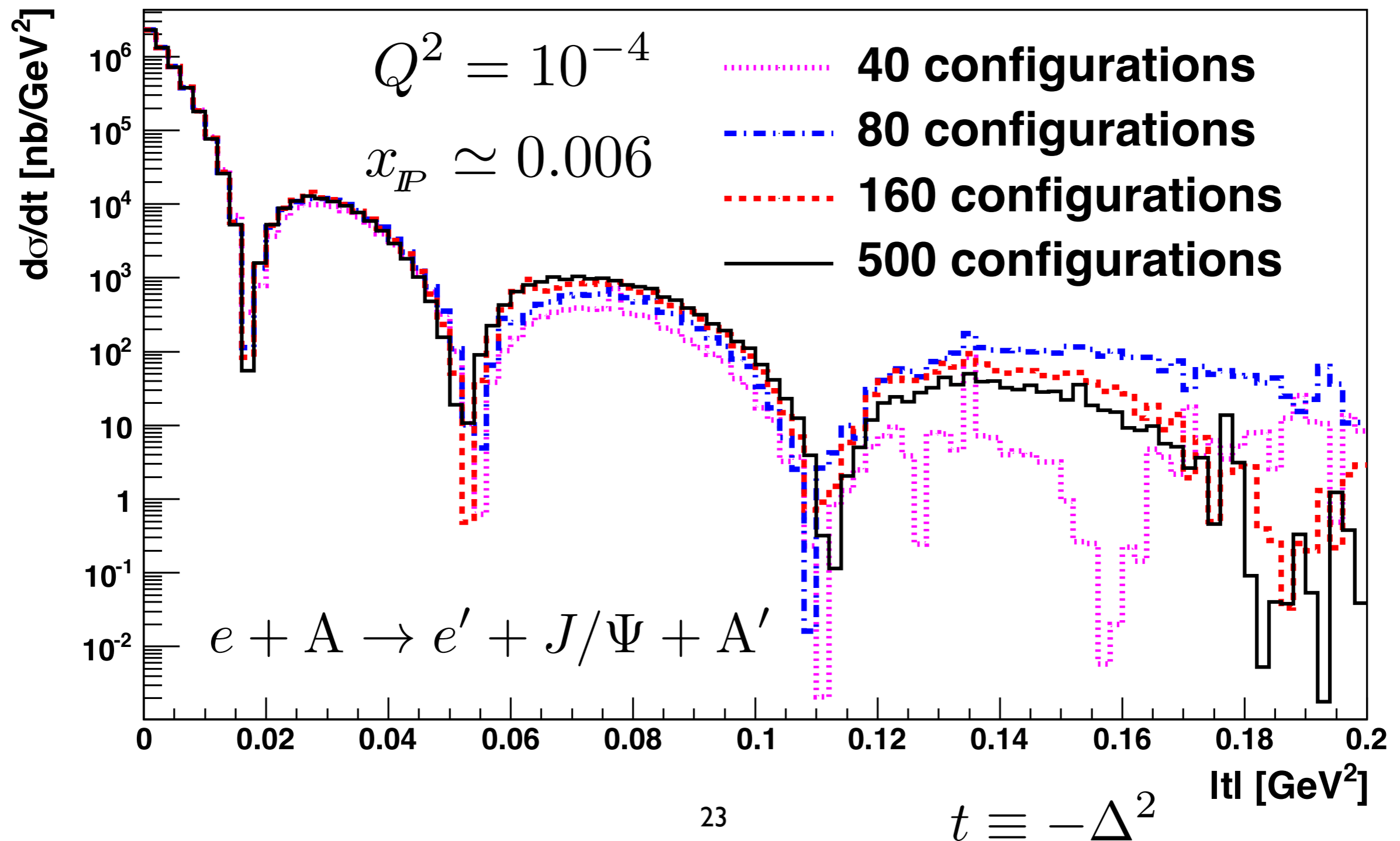
Define average: $\langle \mathcal{O} \rangle_{\Omega} \approx \frac{1}{C_{\text{max}}} \sum_{j=1}^{C_{\text{max}}} \mathcal{O}(\Omega_j)$

$$\mathcal{A}(\Omega_j) = \int dr \frac{dz}{4\pi} d^2\mathbf{b} (\Psi_V^* \Psi)(r, z) 2\pi r b J_0([1 - z]r\Delta) e^{-i\mathbf{b} \cdot \Delta} \frac{d\sigma_{q\bar{q}}}{d^2\mathbf{b}}(x, r, \mathbf{b}, \Omega_j)$$

The question is how many configuration is needed to be averaged over for the cross-section to converge.

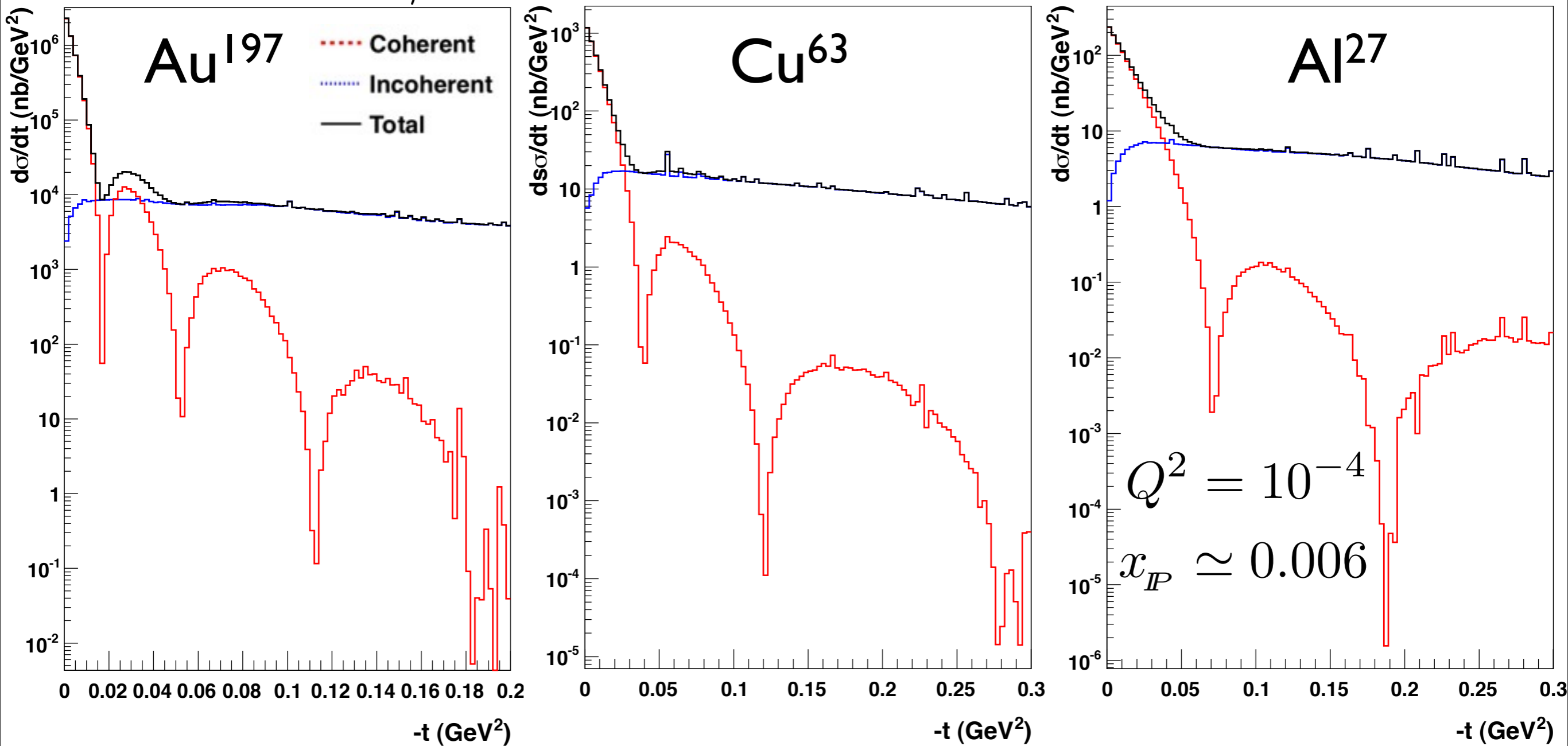
Averaging over initial states

$\Delta = \sqrt{-t}$ is the Fourier conjugate of b . Small variations in b will be seen at large t and vice versa.



Resulting Cross-sections

$$e + A \rightarrow e' + J/\Psi + A'$$



Results are exact within
the bSat model!

Resulting Cross-sections

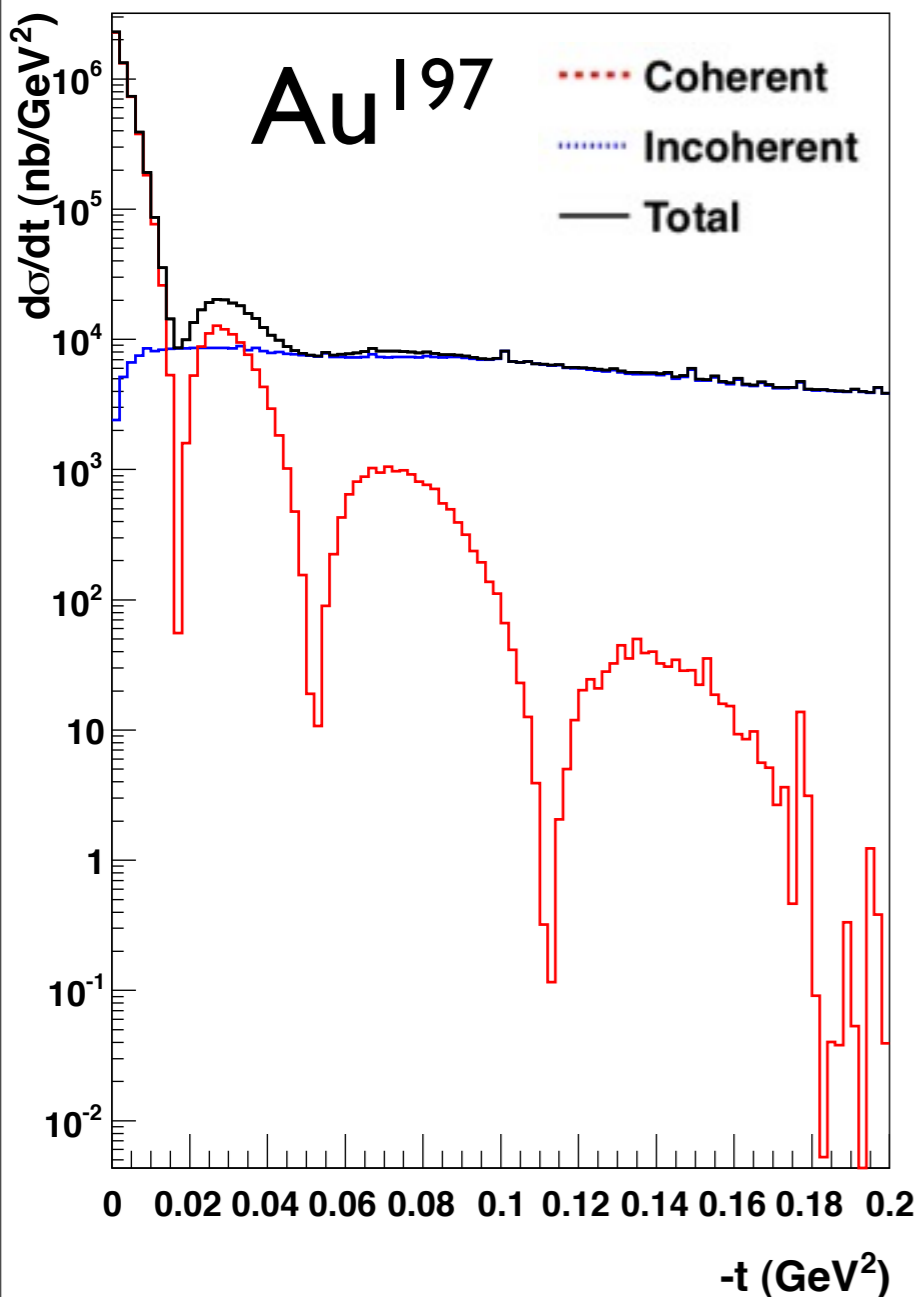
We can suppress the the incoherent background by 3-5 orders of magnitude.

This result means that it will be possible to measure the first 3 coherent bumps and access the spatial gluon distribution at an EIC!!

Incoherent cross-section interesting!!

Has never been measured

Gives access to gluon correlations in the transverse plane



Summary

The bSat and bCGC saturation dipole models have been implemented for vector meson production and DVCS for ep scattering and tested well against HERA data. The models have been extended to eA collisions providing total cross-sections as well as its coherent and incoherent parts for all t , Q^2 and $x < 10^{-2}$.

Outlook

Will publish Sartre 1.0 as soon as it is rewritten.

Sartre 1.1 will include general diffraction:

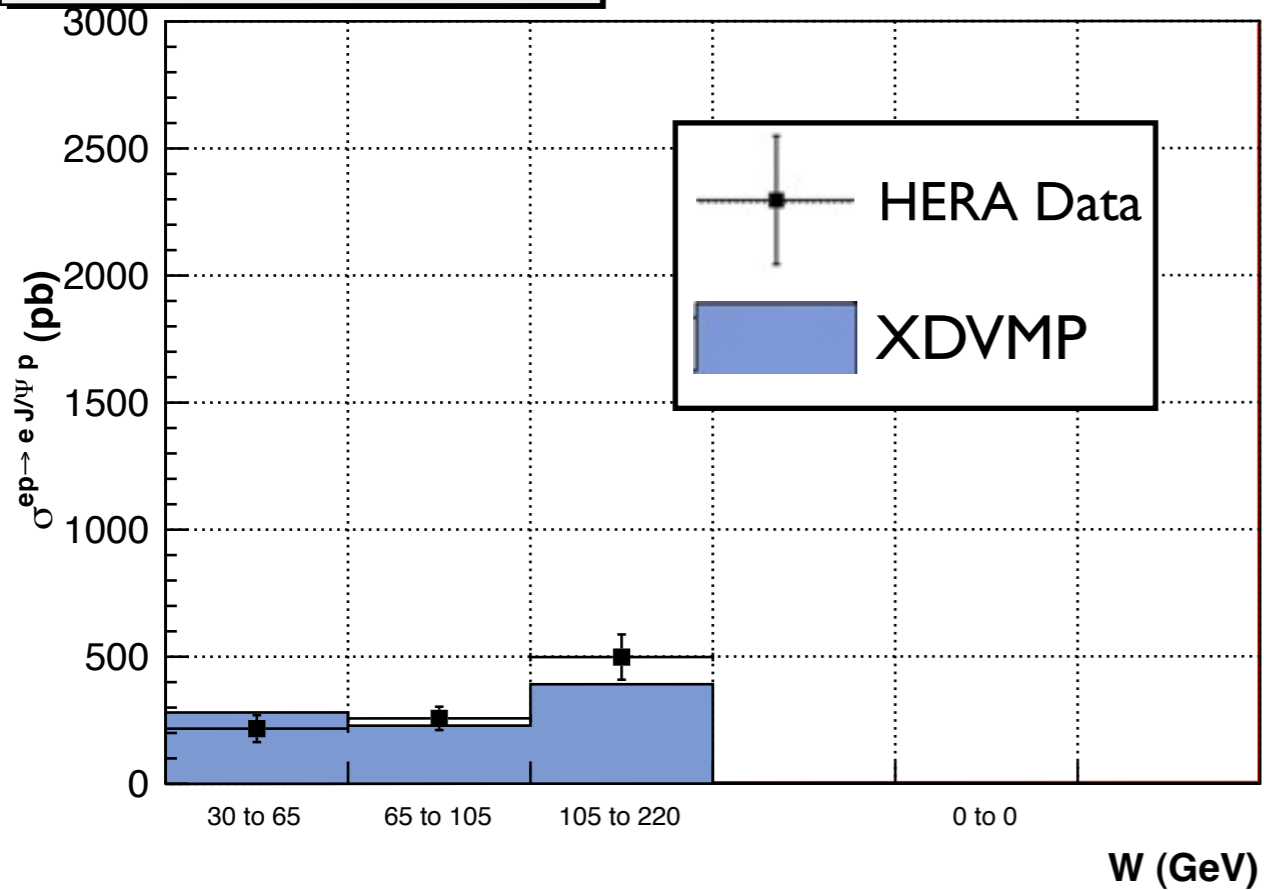
$$e + A \rightarrow e' + X + A'$$

Back up

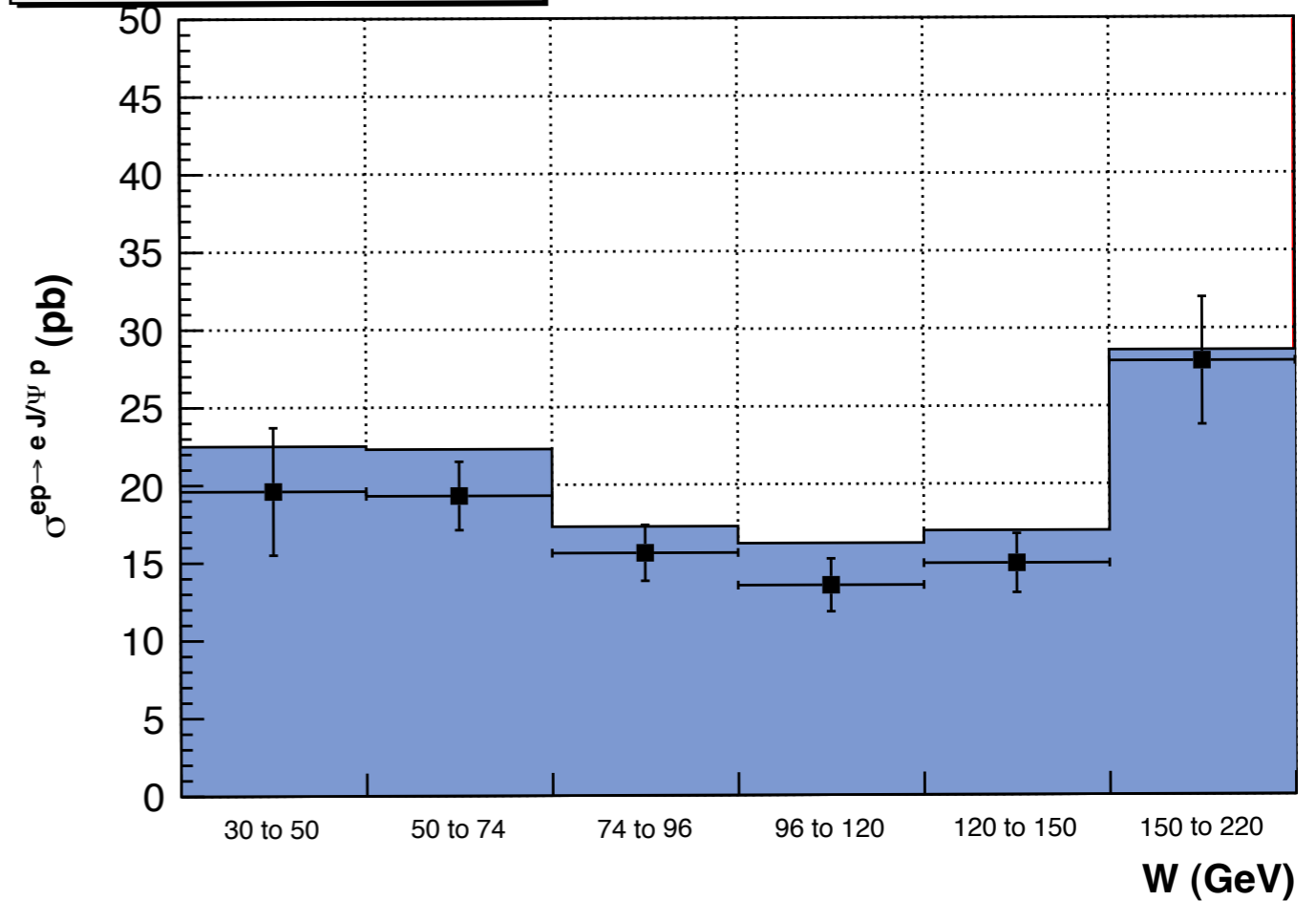
J/Ψ at HERA vs. b-CGC

Exclusive electroproduction of J/Ψ mesons at HERA Nuc. Phys. B695

$0.15 \text{ GeV}^2 < Q^2 < 0.8 \text{ GeV}^2$

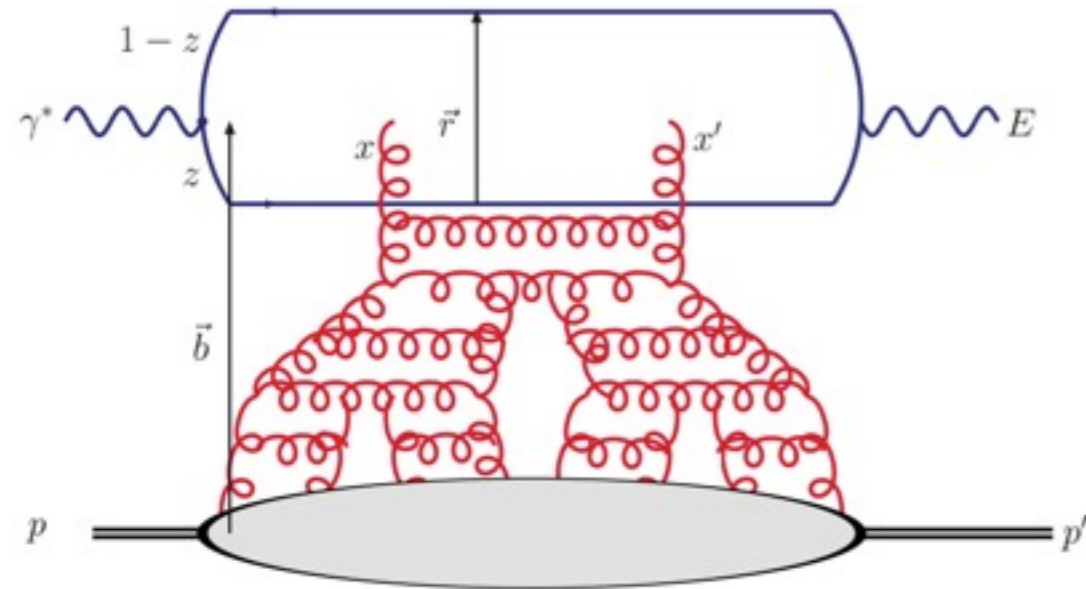


$5 \text{ GeV}^2 < Q^2 < 10 \text{ GeV}^2$



Plots produced by M. Savastio

The b-CGC Model



Includes gluon recombination

$$Y = \ln(1/x), \quad \gamma_s = 0.63, \quad \kappa = 9.9$$

$$\frac{d\sigma_{q\bar{q}}}{d^2\mathbf{b}} = 2 \times \begin{cases} \mathcal{N}_0 \left(\frac{rQ_s}{2} \right)^{2(\gamma_s + \frac{1}{\kappa\lambda Y} \ln \frac{2}{rQ_s})} & rQ_s \leq 2 \\ 1 - e^{-A \ln^2(BrQ_s)} & rQ_s > 2 \end{cases}$$

$$Q_s \equiv Q_s(x, b) = \left(\frac{x_0}{x} \right)^{\lambda/2} \left[\exp \left(-\frac{b^2}{2B_{\text{CGC}}} \right) \right]^{\frac{1}{2\gamma_s}}$$

Generating a Nucleus

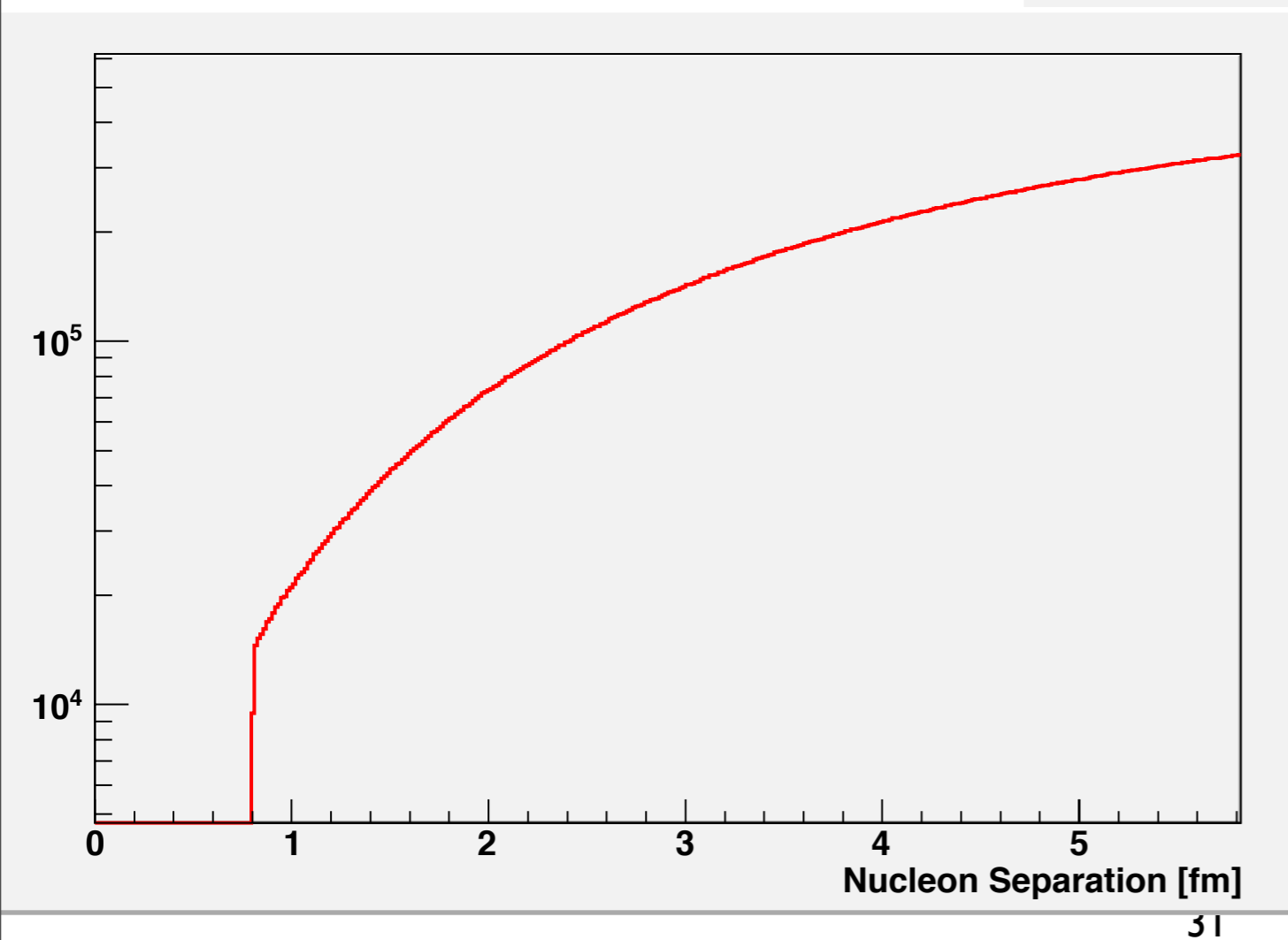
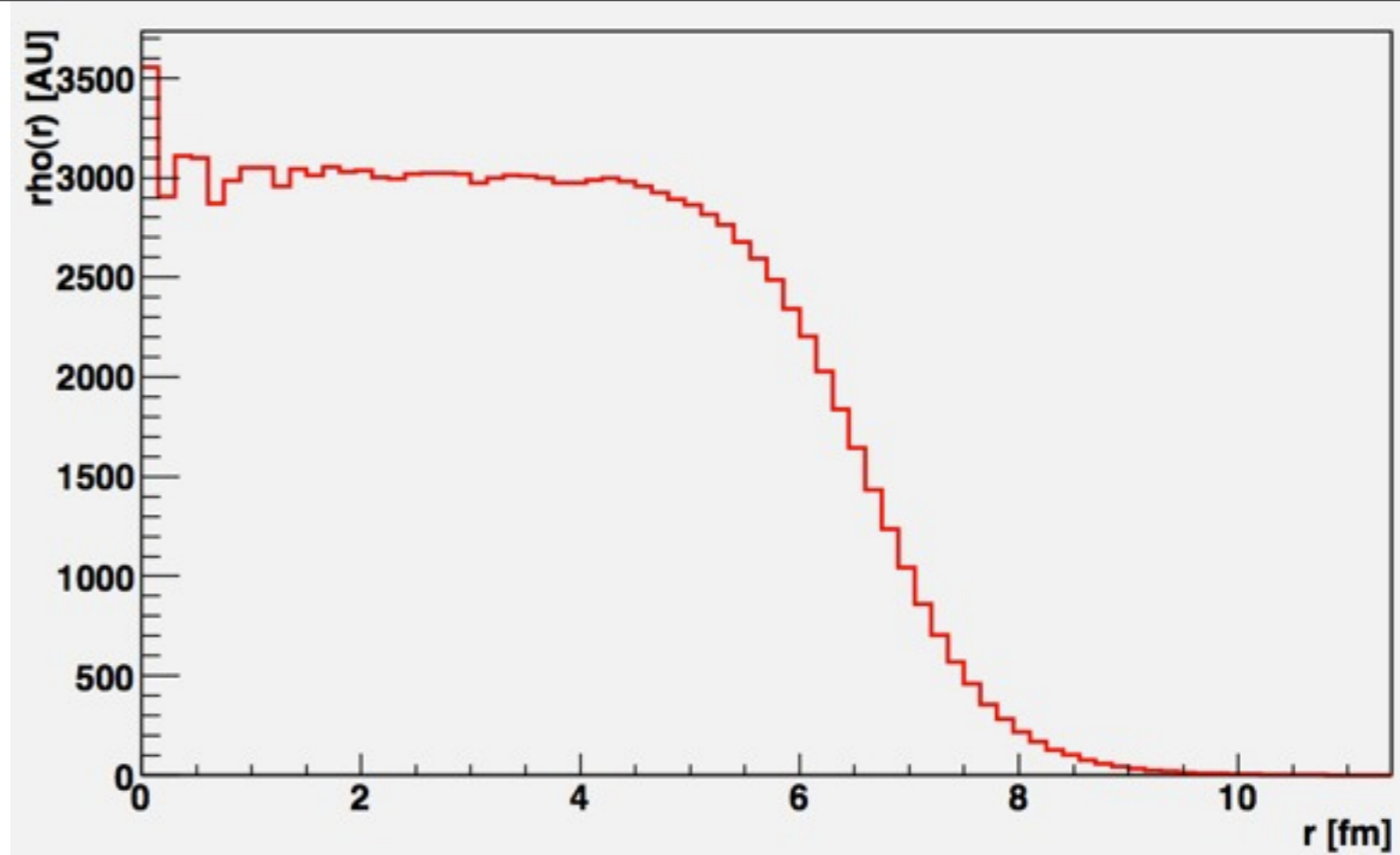
Generate radii according to the Wood-Saxon distribution

$$\rho(r) = \frac{\rho_0}{1 + e^{\frac{r-R_0}{d}}} \quad \rho(r) = \frac{d^3 N}{d^3 \mathbf{r}}$$

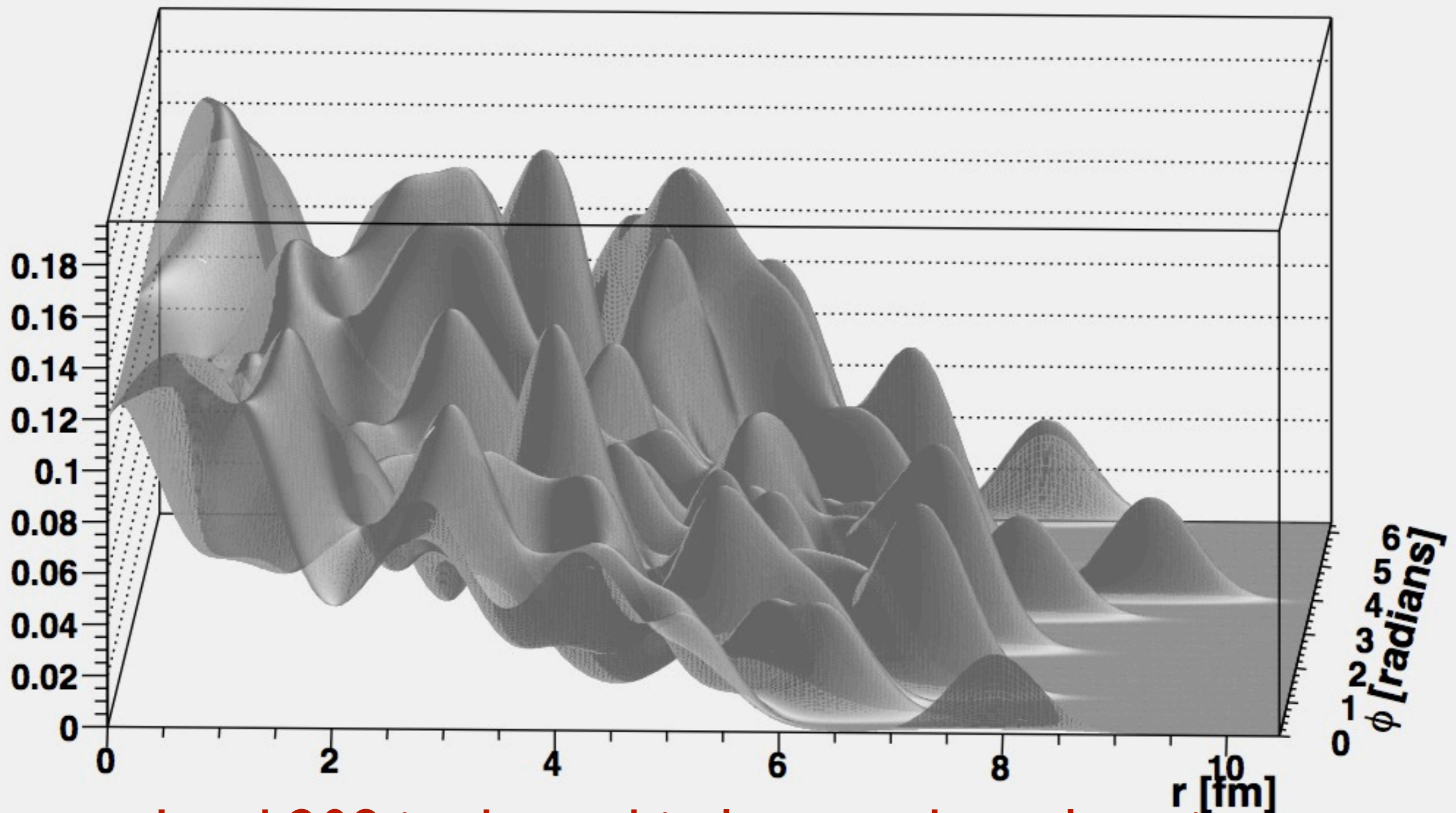
First generate according to r : $\frac{dN}{dr} = 4\pi r^2 \rho(r)$

Then generate angular distributions
uniform in ϕ and $\cos(\theta)$

This is done with a condition that two nucleons can not be within a core distance of $\sim 0.8\text{fm}$.
If they are: regenerate angles (not radius!)



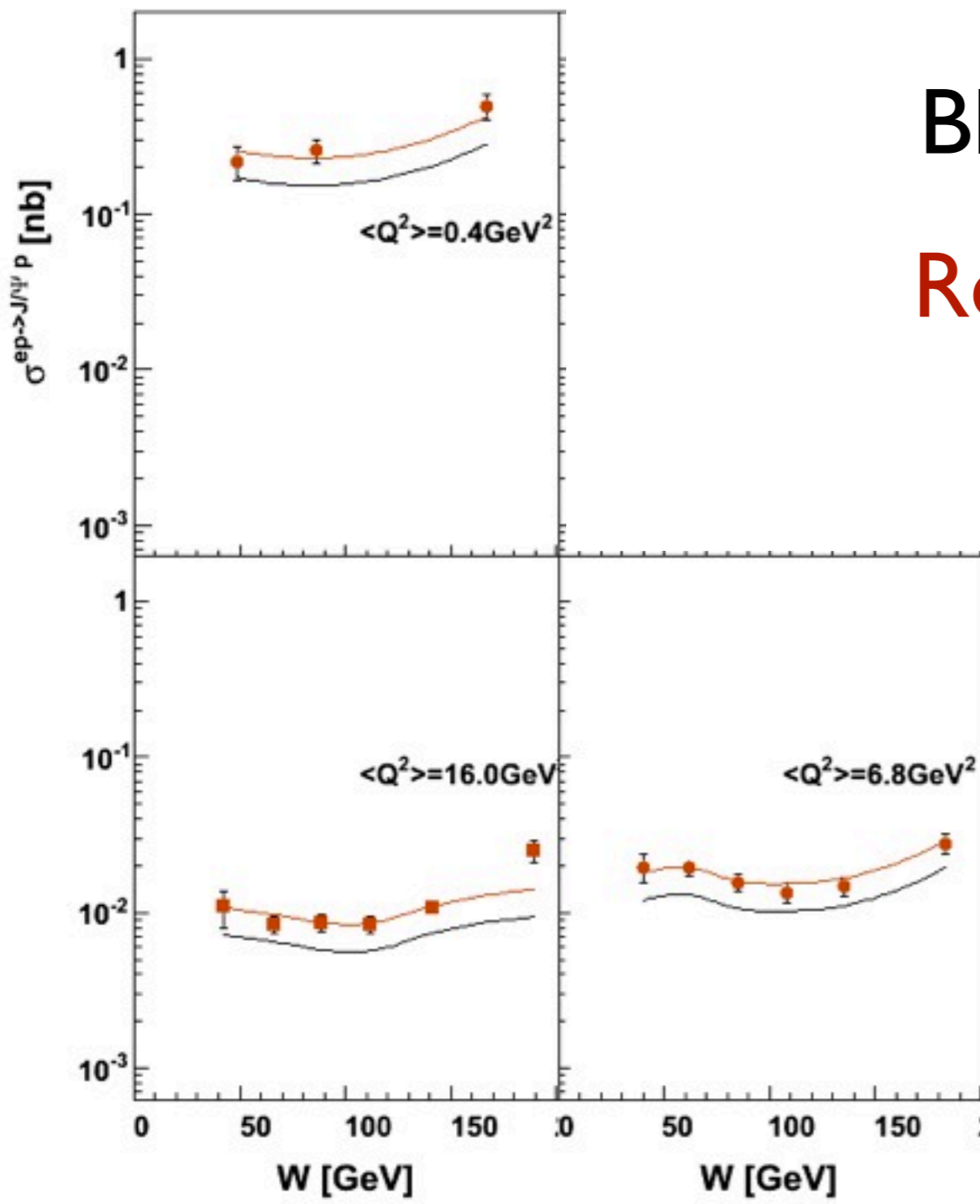
Generating a Nucleus



Lead 208 in the r - ϕ plane, each nucleon is supplemented with a Gaussian width.

First comparison with data

Exclusive electroproduction of J/Psi mesons at HERA Nuc. Phys. B695



Black Curve: XDVMP b-CGC

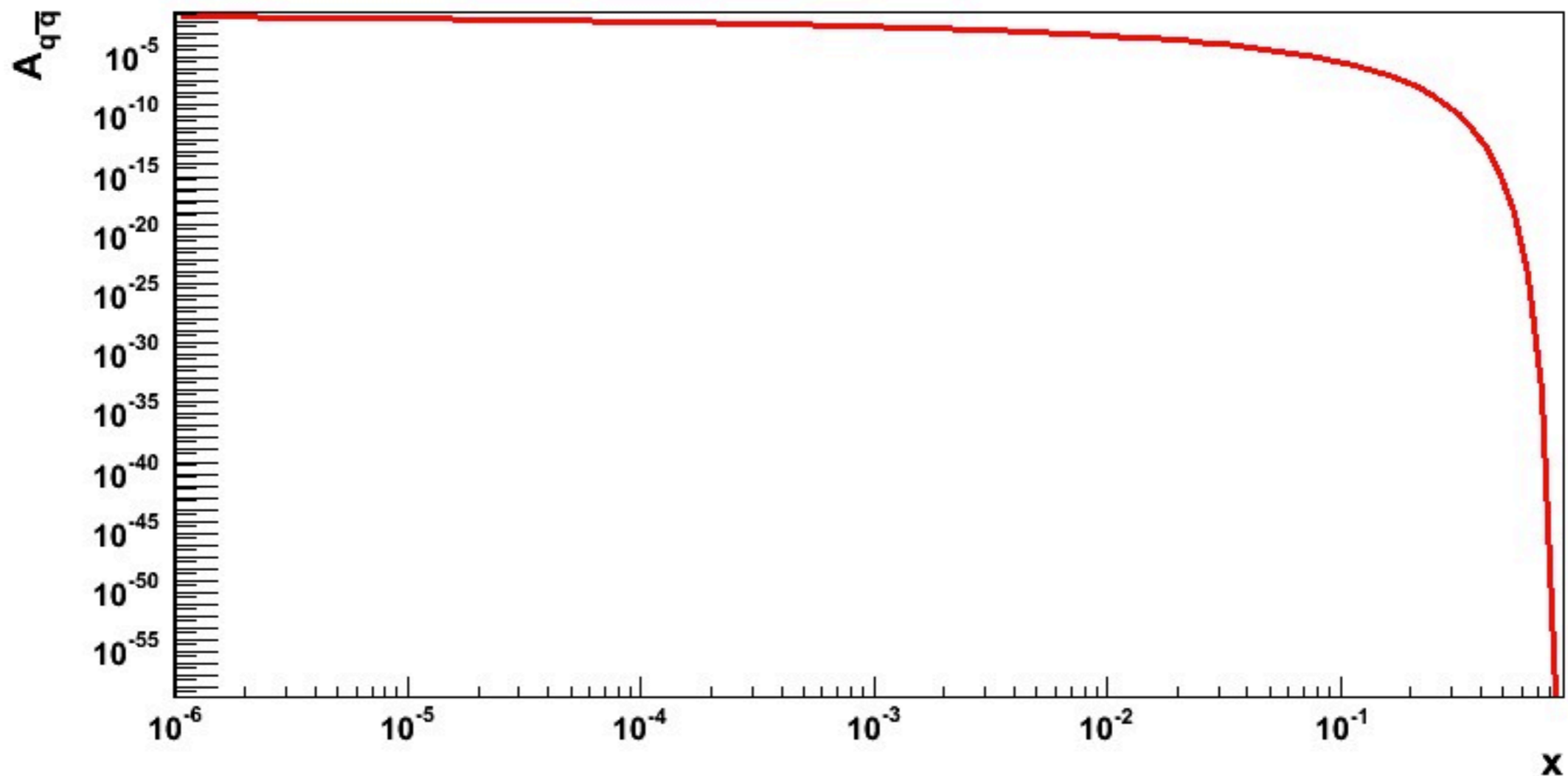
Red Curve: Black Curve x 1.5

Something is missing!!

Plots produced by Ramiro Debbé

Real Amplitude Corrections

$$\beta = \tan(\pi\lambda/2) \qquad \lambda \equiv \frac{\partial \ln \left(\mathcal{A}_{T,L}^{\gamma^* p \rightarrow Ep} \right)}{\partial \ln(1/x)}$$



Real Amplitude Corrections

So far the amplitude has been assumed to be purely imaginary.

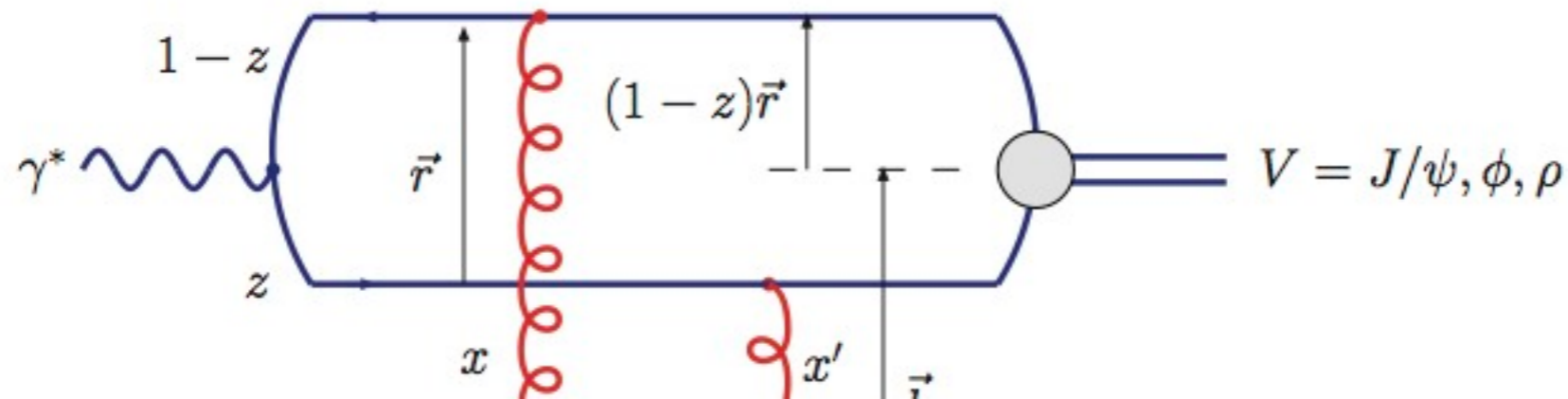
To take the Real part of the amplitude into account it can be multiplied by a factor $(1 + \beta^2)$

β is the ratio Real/Imaginary parts of the Amplitude:

$$\beta = \tan(\pi\lambda/2) \qquad \lambda \equiv \frac{\partial \ln \left(\mathcal{A}_{T,L}^{\gamma^* p \rightarrow Ep} \right)}{\partial \ln(1/x)}$$

This goes bad for large $x \sim 10^{-2}$

Skewedness Corrections



The two gluons carry different momentum fractions

This is the Skewed effect

In leading $\ln(1/x)$ this effect disappears

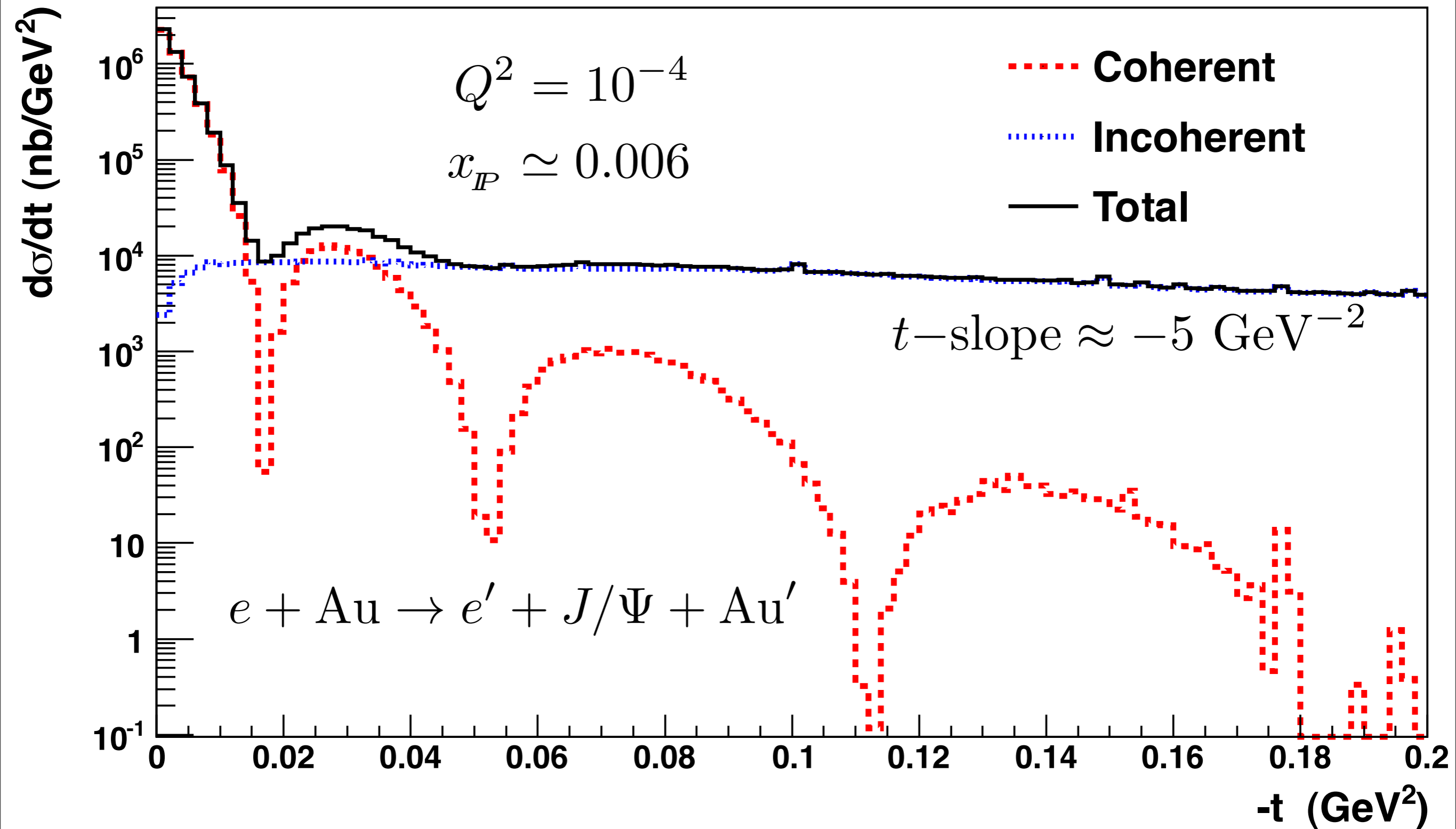
It can be accounted for by a factor R_g

$$R_g(\lambda) = \frac{2^{2\lambda+3}}{\sqrt{\pi}} \frac{\Gamma(\lambda + 5/2)}{\Gamma(\lambda + 4)} \quad \lambda \equiv \begin{cases} \frac{\partial [xg(x, \mu^2)]}{\partial \ln(1/x)} & \text{bSat} \\ \frac{\partial \ln(\mathcal{A}_{T,L}^{\gamma^* p \rightarrow Ep})}{\partial \ln(1/x)} & \text{bCGC} \end{cases}$$

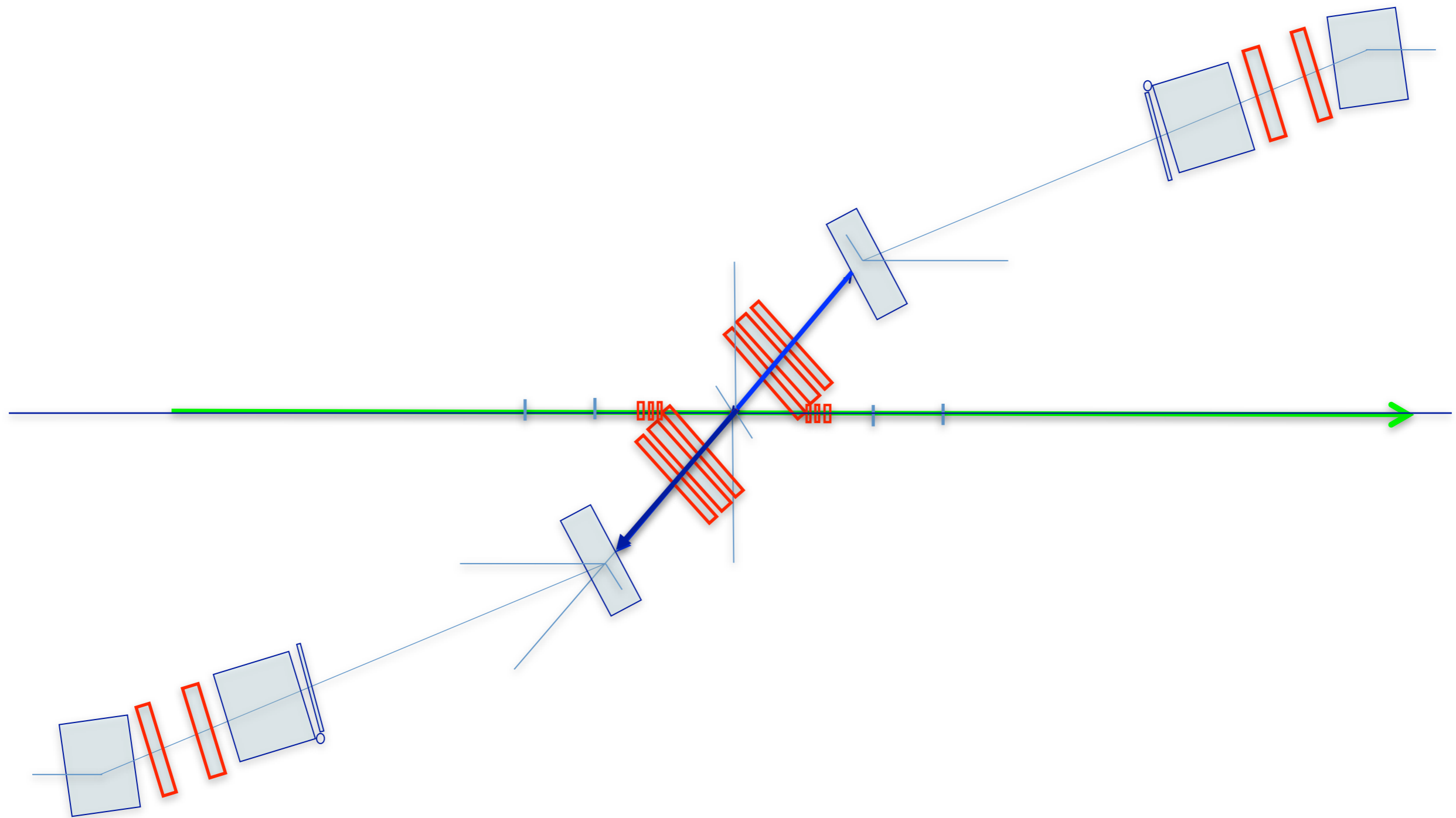
Again, this goes bad for large $x \sim 10^{-2}$!

Implemented with exponential damping to control this.

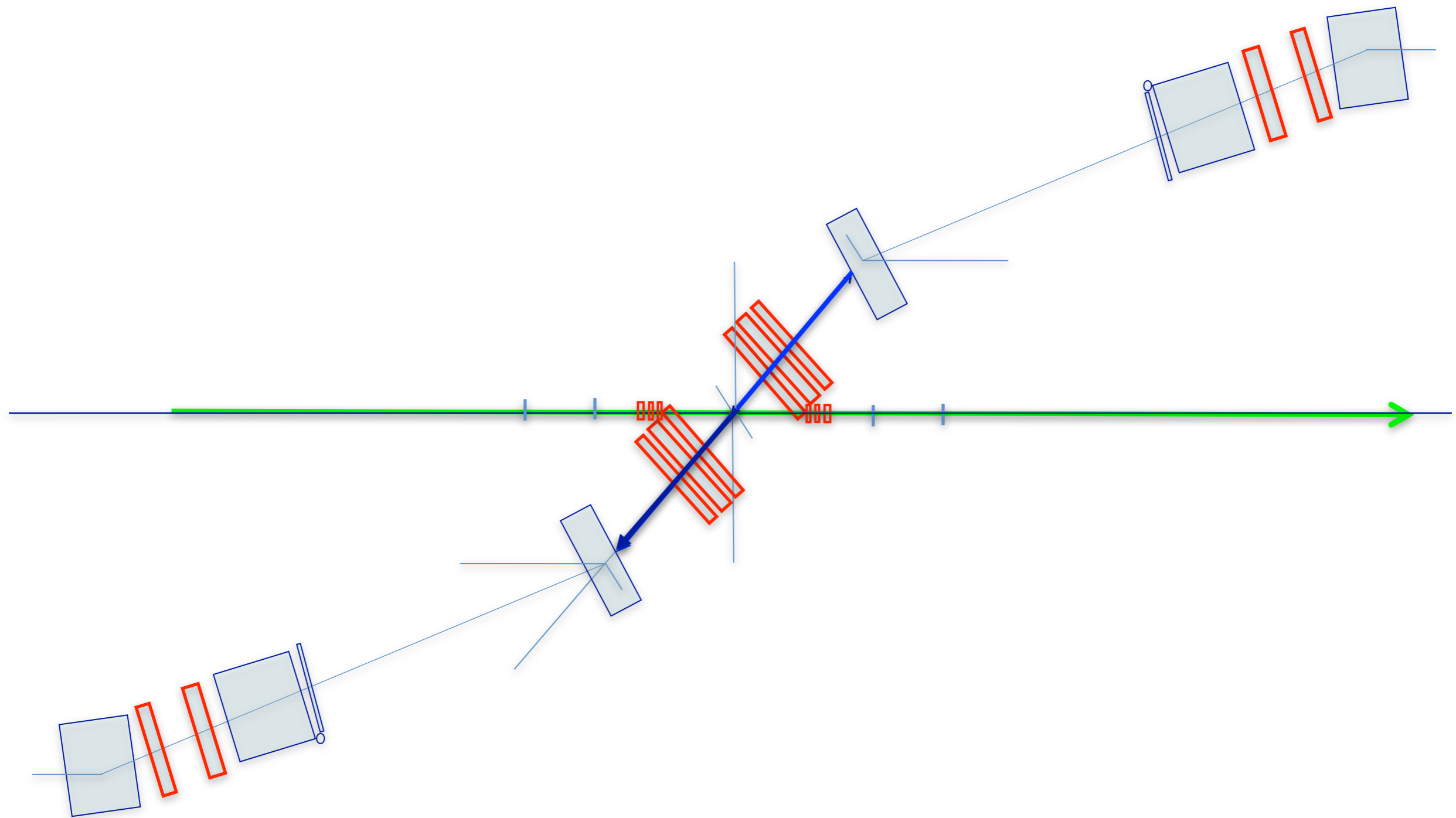
Incoherent/Total



eRHIC IRs, $\beta^*=5\text{cm}$, $l^*=4.5\text{ m}$



eRHIC IRs, $\beta^*=5\text{cm}$, $l^*=4.5\text{ m}$



ep 5x100: $L=3.1 \times 10^{33} \text{cm}^{-2} \text{s}^{-1}$, 200 T/m gradient